Power System Analysis-2 (18EE71)
2021-22

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Syllabus

B. E. ELECTRICAL AND ELECTRONICS ENGINEERING **Choice Based Credit System (CBCS) and Outcome Based Education (OBE)** **SEMESTER – VII**			
POWER SYSTEM ANALYSIS – 2(Core Course)			
Course Code	**18EE71**	CIE Marks	40
Number of Lecture Hours/Week	2:2:0	SEE Marks	60
Credits	03	Exam Hours	03

Course Learning Objectives:

- To explain formulation of network models and bus admittance matrix for solving load flow problems.
- To discuss optimal operation of generators on a bus bar and optimum generation scheduling.
- To explain symmetrical fault analysis and algorithm for short circuit studies.
- To explain formulation of bus impedance matrix for the use in short circuit studies on power systems.
- To explain numerical solution of swing equation for multi-machine stability

Module-1

Network Topology: Introduction and basic definitions of Elementary graph theory Tree, cut-set, loop analysis. Formation of Incidence Matrices. Primitive network- Impedance form and admittance form, Formation of Y Bus by Singular Transformation. Y_{bus} by Inspection Method. Illustrative examples. T1,2

Module-2

Load Flow Studies: Introduction, Classification of buses. Power flow equation, Operating Constraints, Data for Load flow, Gauss Seidal iterative method. Illustrative examples. T1, R1

Module-3

Load Flow Studies(continued) Newton-Raphson method derivation in Polar form, Fast decoupled load flow method, Flow charts of LFS methods. Comparison of Load Flow Methods. Illustrative examples. T1, R1

Module-4

Economic Operation of Power System: Introduction and Performance curves Economic generation scheduling neglecting losses and generator limits Economic generation scheduling including generator limits and neglecting losses Economic dispatch including transmission losses Derivation of transmission loss formula. Illustrative examples.T1

Unit Commitment: Introduction, Constraints and unit commitment solution by prior list method and dynamic forward DP approach (Flow chart and Algorithm only). T3

Module-5

Symmetrical Fault Analysis: Z Bus Formulation by Step by step building algorithm without mutual coupling between the elements by addition of link and addition of branch. Illustrative examples. Z bus Algorithm for Short Circuit Studies excluding numerical.T1

Power System Stability: Numerical Solution of Swing Equation by Point by Point method and Runge Kutta Method. Illustrative examples. T1

Course Outcomes: At the end of the course the student will be able to:

- Formulate network matrices and models for solving load flow problems.
- Perform steady state power flow analysis of power systems using numerical iterative techniques.
- Solve issues of economic load dispatch and unit commitment problems.
- Analyze short circuit faults in power system networks using bus impedance matrix.
- Apply Point by Point method and Runge Kutta Method to solve Swing Equation.

Question paper pattern:

- The question paper will have ten questions.
- Each full question is for 20 marks.
- There will be 2 full questions (with a maximum of three sub questions in one full question) from each module.
- Each full question with sub questions will cover the contents under a module.
- Students will have to answer 5 full questions, selecting one full question from each module.
  Module 1 Y Bus Matrix size limited to 3X3 for illustrative examples.
  Module 2 NR Method limited to 3 bus system with one iteration for illustrative examples.

Text Books

1	Modern Power System Analysis	D P Kothari, I J Nagrath	McGraw Hill	4th Edition, 2011
2	Computer Methods in Power Systems Analysis	Glenn W. Stagg Ahmed H Ei - Abiad	Scientific International Pvt. Ltd.	1st Edition, 2019
3	Power Generation Operation and Control	Allen J Wood etal	Wiley	2nd Edition,2016

Reference Books

1	Computer Techniques in Power System Analysis	M.A. Pai	McGraw Hill	2nd Edition, 2012
2	Power System Analysis	Hadi Saadat	McGraw Hill	2ndEdition, 2002
3	Computer Techniques and Models in Power System Analysis	K. Uma Rao	IK International	2013

Course Outcomes

CO1	Formulate network matrices and models for solving load flow problems.
CO2	Perform power flow analysis of power systems using numerical iterative techniques
CO3	Solve issues of economic load dispatch and unit commitment problems
CO4	Analyze short circuit faults in power system networks using bus impedance matrix
CO5	Apply numerical techniques to solve swing equation for stability analysis.

CO-PO-PSO articulation Matrix

COs	**POs**											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
CO1	3	3										
CO2	3	3			2							
CO3	3	2			2		1					
CO4	3	2										
CO5	3	2				1						

COs	**PSOs**		
	1	**2**	**3**
CO1	3	2	
CO2	3	2	
CO3	3	2	
CO4	3	2	
CO5	3	2	

Module 1

Network Topology

Table of Contents

1.1. Introduction

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents. The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix. The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems. The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

1.2. Elementary Linear Graph Theory: Important Terms

The geometrical interconnection of the various branches of a network is called the topology of the network. The connection of the network topology, shown by replacing all its elements by lines is called a graph. A linear graph consists of a set of objects called nodes and another set called elements such that each element is identified with an ordered pair of nodes. An element is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a direction is assigned to each element is called an oriented graph or a directed graph. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph: This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1.1, with:

e = number of elements = 6

n = number of nodes = 4

b = number of branches = n-1 = 3

l = number of links = e-b = 3

Tree = T(1,2,3) and

Co-tree = T(4,5,6)

Sub-graph: Sg is a sub-graph of G if the following conditions are satisfied:

· Sg is itself a graph

· Every node of Sg is also a node of G

· Every branch of Sg is a branch of G

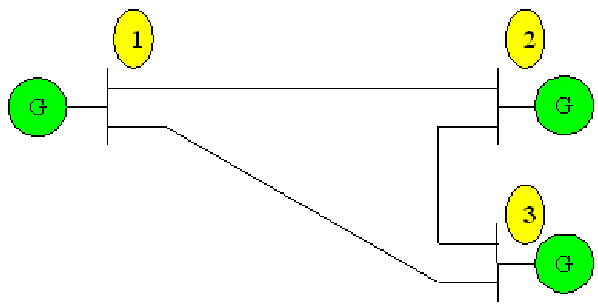


Fig 1.1: Single line diagram of a power system

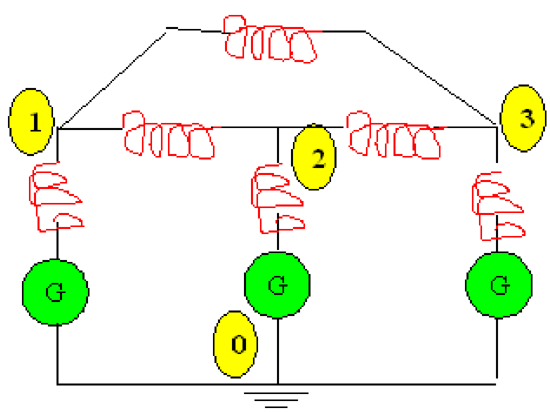


Fig 1.2: Reactance diagram of the given power system in fig 1.1

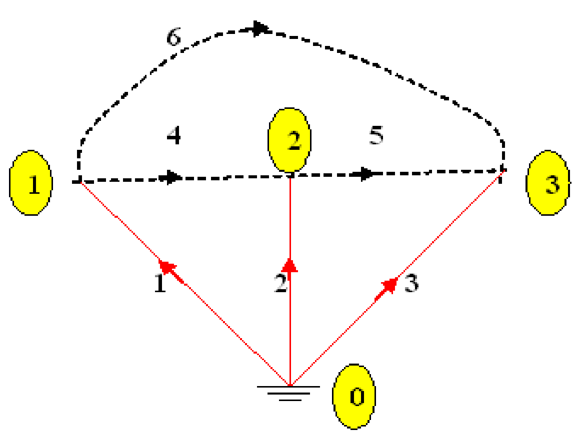


Fig 1.3: Oriented graph

Cutset: It is a set of branches of a connected graph G which satisfies the following conditions:

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1.3, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected subgraphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

The number of branches: b = n-1 **(1)**

For the graph of Fig 1.3, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the ***co-tree***. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called ***links***. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

The number of links: $\boldsymbol{l} = e - b = e - n + 1$ **(2)**

For the graph of Fig 1.3, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

Examples

Example-1: Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.

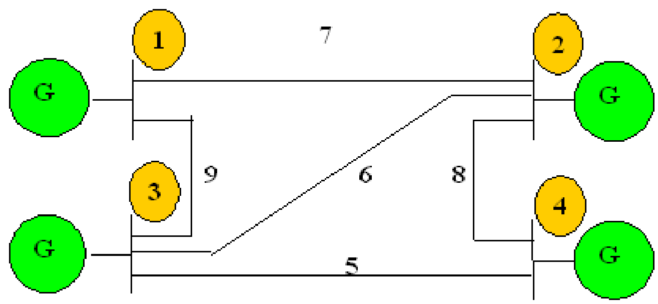


Fig. E1a. Single line diagram of Example System

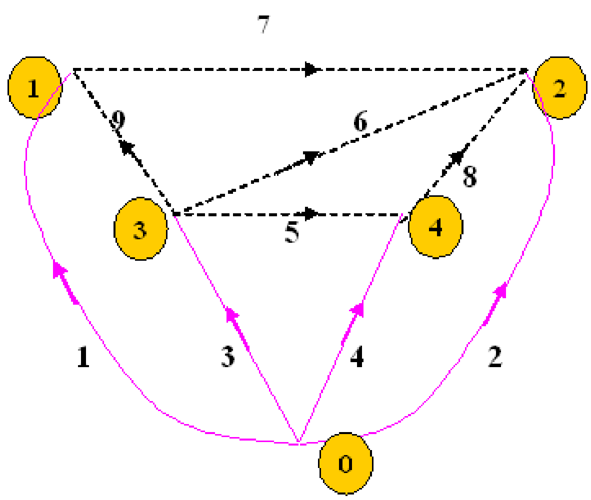


Fig. E1b. Oriented Graph of Fig. E1a.

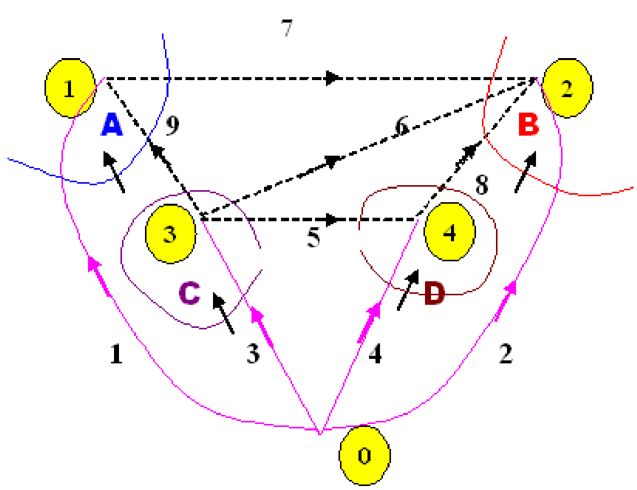


Fig. E1c. Basic Cutsets of Fig. E1a.

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cutsets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figure E1c and Fig.E1d respectively.

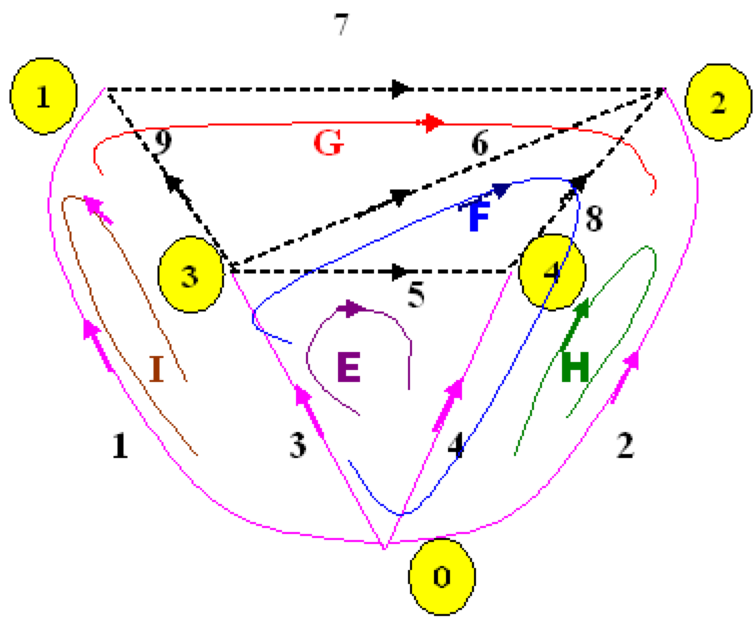


Fig. E1d. Basic Loops of Fig. E1a.

1.3. Incidence Matrices

Element–node incidence matrix: **Aˆ**

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, $A\hat{}$.

An element aij of $A\hat{}$ is defined as under:

aij = 1 if the branch-i is incident to and oriented away from the node-j.

= -1 if the branch-i is incident to and oriented towards the node-j.

= 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of $A\hat{}$ is e×n, where e is the number of elements and n is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1.3 the corresponding element-node incidence matrix, is obtained as under:

$\hat{A}$ =

Nodes / Elements	0	1	2	3
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than *n*. Thus in general, the matrix *A*ˆ satisfies the identity:

$$\sum_{j=1}^{n} \mathbf{a_{ij} = 0} \quad \forall \;\; i = 1,2,\ldots..e. \qquad \textbf{(3)}$$

Bus incidence matrix: A

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from *A*ˆ to obtain the bus incidence matrix, A. The dimensions of A are e *(n-1)* and the rank is *n-1*. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0, as under:

A =

Buses / Elements	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices Ab and Al as shown, where,

(i) Ab is of dimension (bxb) corresponding to the branches and
(ii) Al is of dimension (lxb) corresponding to links.

A is a rectangular matrix, hence it is singular. Ab is a non-singular square matrix of dimension-b. Since A gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

$$AT\, i = 0 \qquad \textbf{(4)}$$

where AT is the transpose of matrix A and *i* is the vector of branch currents. Similarly for the branch voltages we can write,

$$v = A\ bus\ E \qquad \textbf{(5)}$$

Examples on Bus Incidence Matrix:

Example-2: For the sample network-oriented graph shown in Fig. E2, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and Aˆ . Also show the partitioned form of the matrix-A.

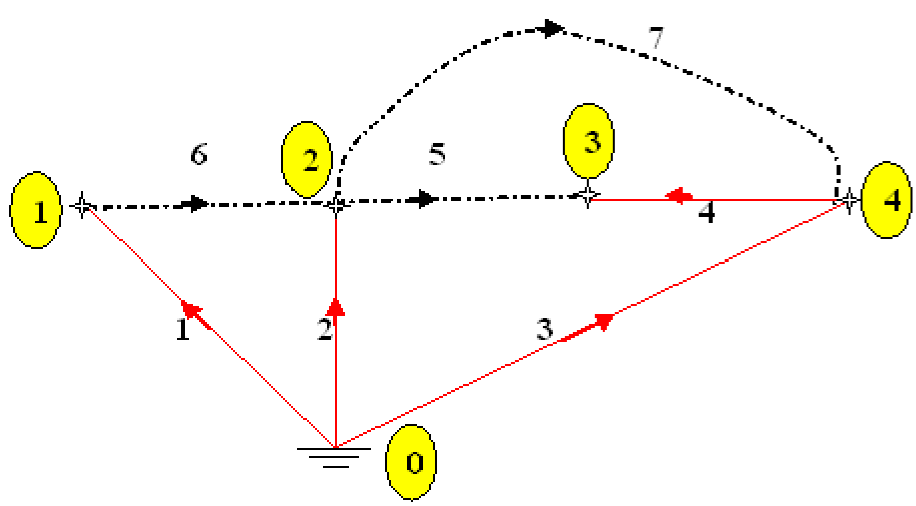


Fig. E2. Sample Network-Oriented Graph

$$\hat{A} = \text{Elements} \overset{\text{nodes}}{\begin{bmatrix} e\backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

$$\mathbf{A} = \text{Elements} \overset{\text{buses}}{\begin{bmatrix} e\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$$A_b = \text{branches} \overset{\text{buses}}{\begin{bmatrix} b\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix}}$$

$$A_l = \text{links} \overset{\text{buses}}{\begin{bmatrix} l\backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Example-3: For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree,

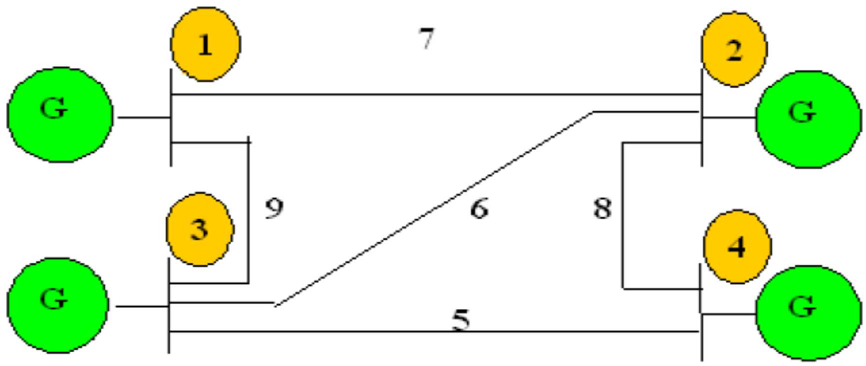


Fig. E3a. Sample Example network

T(1,2,3,4), obtain the incidence matrices A andAˆ . Also show the partitioned form of the matrix-A.

Consider the oriented graph of the given system as shown in figure E3b, below.

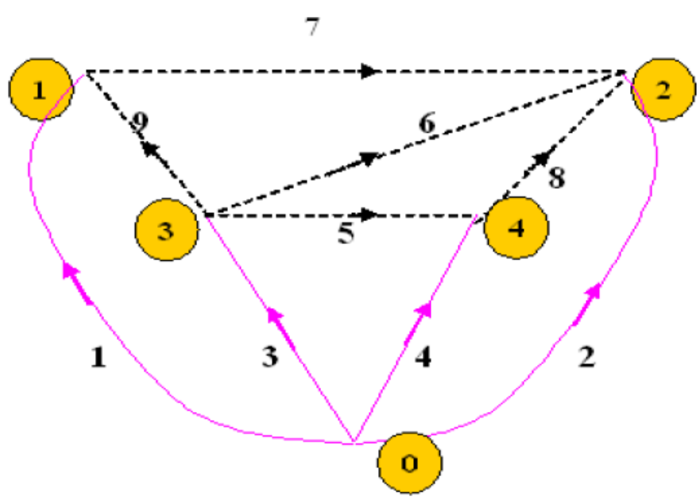


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices • and A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$A_b =$

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

$A_l =$

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

1.4. Primitive Networks

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

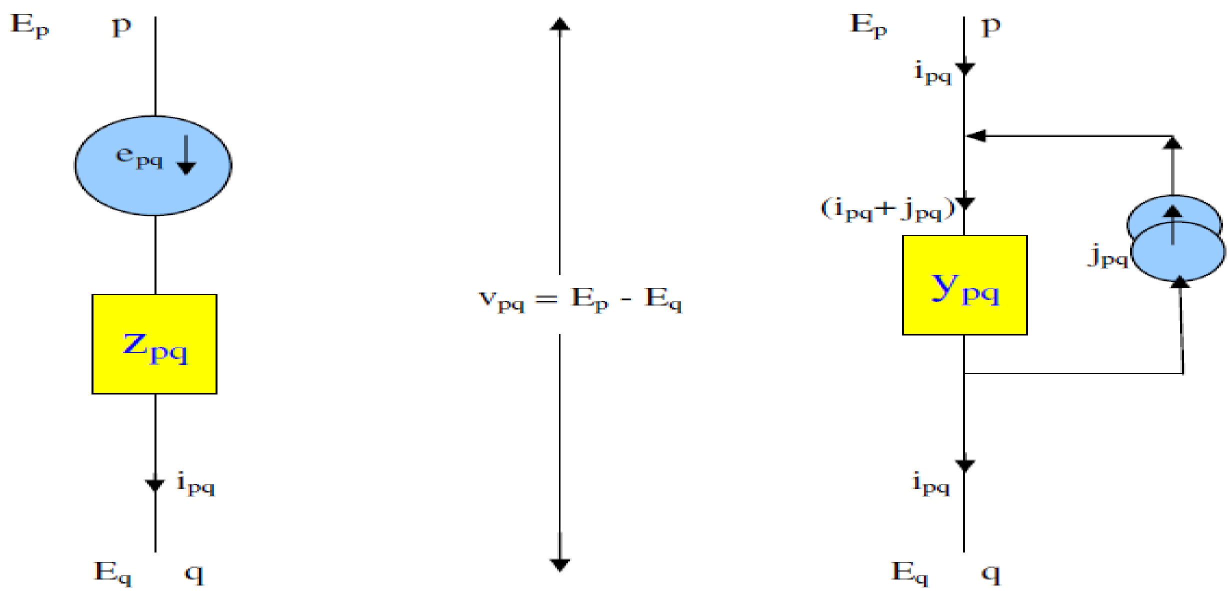


Fig.2 Representation of a primitive network element (a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

vpq = voltage across the element p-q,

epq = source voltage in series with the element p-q, ipq= current through the element p-q,

jpq= source current in shunt with the element p-q, zpq= self impedance of the element p-q and

ypq= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, Vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

vpq + epq = zpqipq (in its impedance form)

ipq + jpq = ypqvpq (in its admittance form) **(6)**

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:

jpq = - ypq epq **(7)**

A set of non-connected elements of a given system is defined as a *primitive Network and a*n element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

v + e = [z] i

i + j = [y] v **(8)**

Primitive network matrices:

A diagonal element in the matrices, [z] or [y] is the self impedance zpq-pq or self admittance, ypq-pq. An off-diagonal element is the mutual impedance, zpq-rs or mutual admittance, ypq-rs, the value present as a mutual coupling between the elements p-q and r-

s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Examples on Primitive Networks:

Example-4: Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: Z1=Z2=0.2, Z3=0.25, Z4=Z5=0.1 and Z6=0.4 units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Solution:

The element node incidence matrix, $A\hat{}$ can be obtained from the given A matrix, by pre- augmenting to it an extra column corresponding to the reference node, as under.

$\hat{A}$ **=**

1	-1	0	0
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	0	1	-1
0	1	0	-1

Based on the conventional definitions of the elements of $A\hat{}$, the oriented graph can be formed as under:

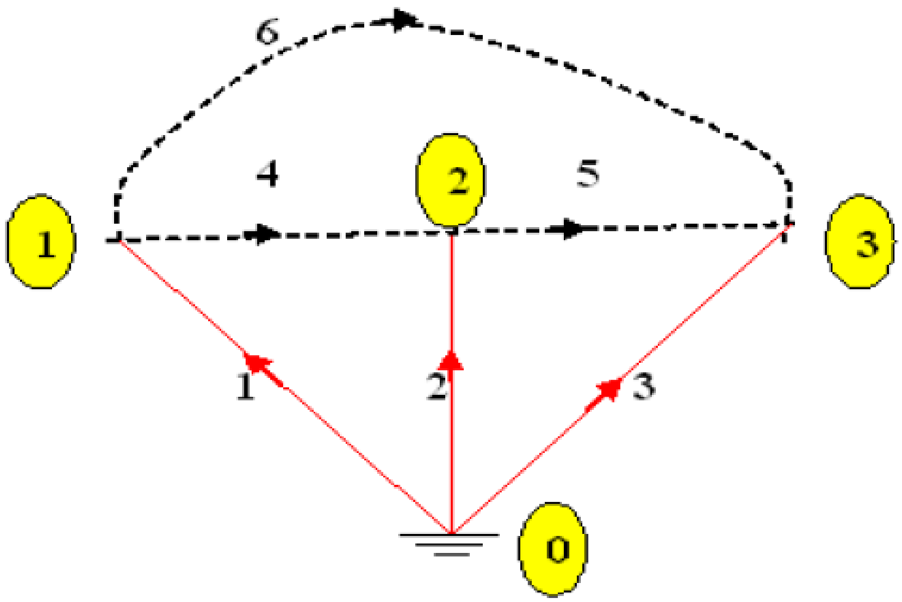


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

[z] =

0.2	0	0	0	0	0
0	**0.2**	0	0	0	0
0	0	**0.25**	0	0	0
0	0	0	**0.1**	0	0
0	0	0	0	**0.1**	0
0	0	0	0	0	**0.4**

And

[y] =

5.0	0	0	0	0	0
0	**5.0**	0	0	0	0
0	0	**4.0**	0	0	0
0	0	0	**10**	0	0
0	0	0	0	**10**	0
0	0	0	0	0	**2.5**

Example-5: Consider three passive elements whose data is given in Table E5 below. Form the primitive network impedance matrix.

Element number	Self impedance (z_{pq-pq})		Mutual impedance, (z_{pq-rs})	
	Bus-code, (p-q)	Impedance in p.u.	Bus-code, (r-s)	Impedance in p.u.
1	1-2	j 0.452		
2	2-3	j 0.387	1-2	j 0.165
3	1-3	j 0.619	1-2	j 0.234

Solution:

$$[z] = \begin{array}{c} \\ 1\text{-}2 \\ 2\text{-}3 \\ 1\text{-}3 \end{array} \begin{array}{c} \begin{array}{ccc} 1\text{-}2 & 2\text{-}3 & 1\text{-}3 \end{array} \\ \left[\begin{array}{ccc} j\,0.452 & j\,0.165 & j\,0.234 \\ j\,0.165 & j\,0.387 & 0 \\ j\,0.234 & 0 & j\,0.619 \end{array}\right] \end{array}$$

Note:

· The size of [z] is e´ e, where e= number of elements,

· The diagonal elements are the self impedances of the elements

· The off-diagonal elements are mutual impedances between the corresponding elements.

· Matrices [z] and [y] are inter-invertible.

1.5. Formation of Ybus And Zbus

The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. ZBUS Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

EBUS = ZBUS IBUS

IBUS = YBUS EBUS **(9)**

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

EBR = ZBR IBR

IBR = YBR EBR **(10)**

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

ELOOP = ZLOOP ILOOP

ILOOP = YLOOP ELOOP **(11)**

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

1.5.1. Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation:

I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)

At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2) **(12)**

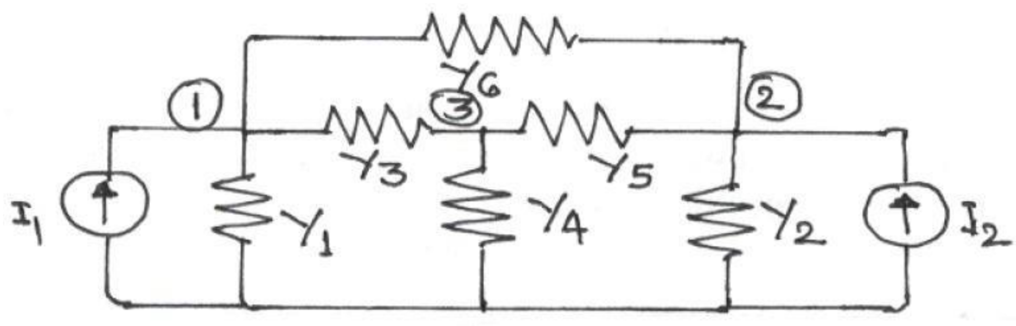


Fig. 3 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \quad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form

IBUS = YBUS EBUS **(14)**

Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively. By observing the elements of the bus admittance matrix, YBUS of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i, *Off Diagonal elements*: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)

Yij = - yij (j = 1,2,…….n) **(15)**

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix:

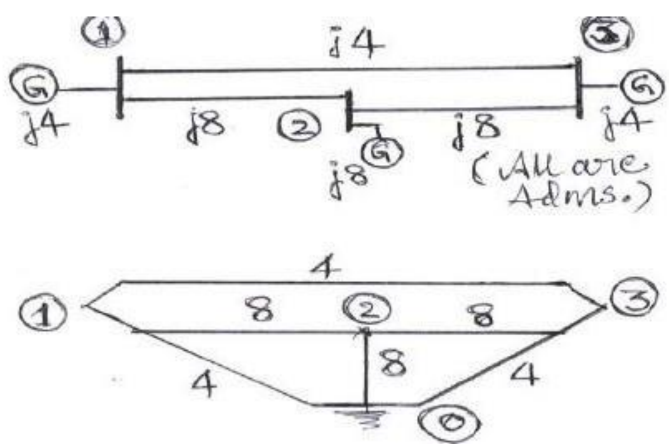
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are interinvertible.

Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

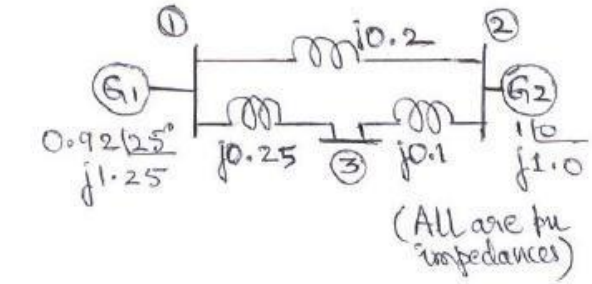
Examples on Rule of Inspection:

Example 6: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

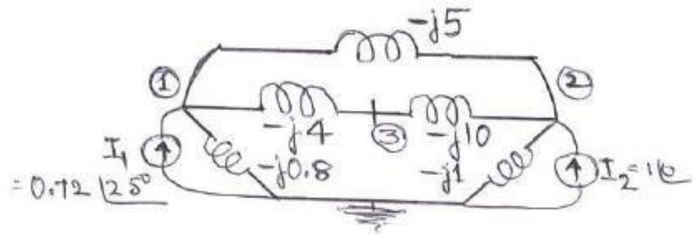
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



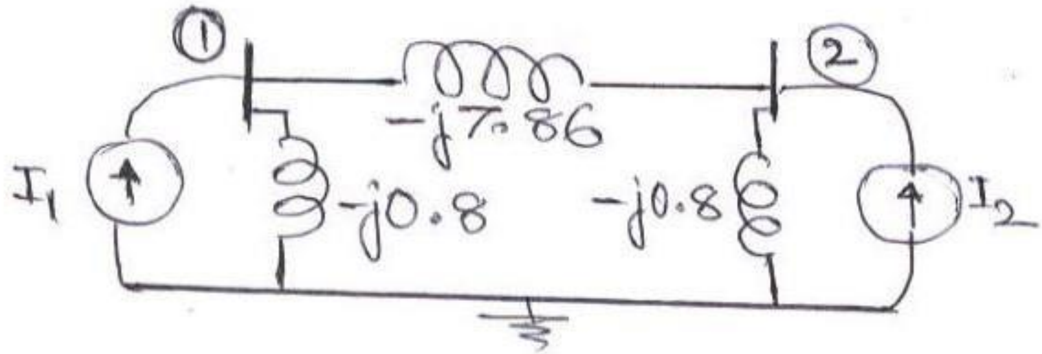
Example 7: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

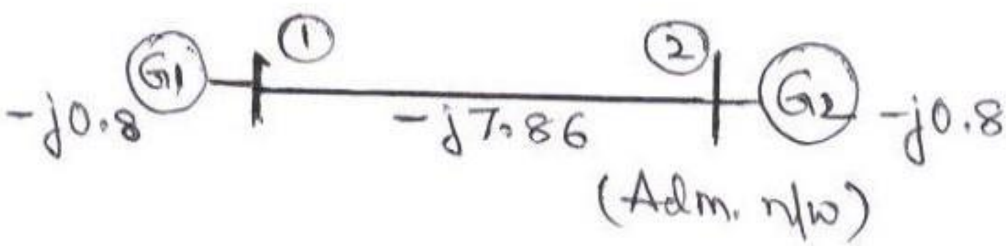


$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$



$$Z_{BUS} = Y_{BUS}^{-1}$$





$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

1.5.2. Singular Transformations

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).

For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

IBUS = YBUS EBUS **(17)**

Where EBUS = vector of bus voltages measured with respect to reference bus IBUS = Vector of currents injected into the bus

YBUS = bus admittance matrix

The performance equation of the primitive network in admittance form is given by i + j = [y] v

Pre-multiplying by At (transpose of A), we obtain

At i +At j = At [y] v **(18)**

However, as per equation (4),

At i =0,

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

At j = IBUS **(19)**

Thus from (18) we have, IBUS = At [y] v **(20)**

However, from (5), we have v =A EBUS

And hence substituting in (20) we get,

IBUS = At [y] A EBUS **(21)**

Comparing (21) with (17) we obtain,

YBUS = At [y] A **(22)**

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

ZBUS = $YBUS^{-1}$ **(23)**

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.

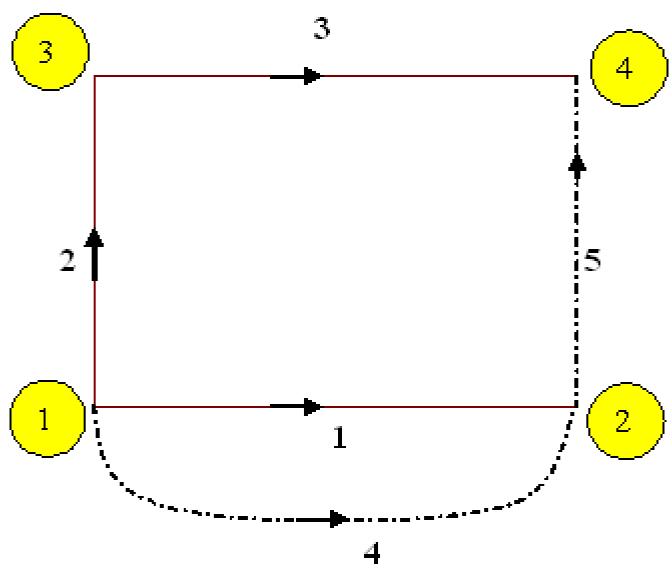


Fig E8 System for Example-8 Table E8: Data for Example

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix [y] = [z]-1 and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

Power System Analysis-2 (18EE71)
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Module 2

Load Flow Studies

Table of Contents

2.1. Introduction

Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

- The Kirchhoff's relations holding good,
- Capability limits of reactive power sources,
- Tap-setting range of tap-changing transformers,
- Specified power interchange between interconnected systems,
- Selection of initial values, acceleration factor, convergence limit, etc.

2.2. Classification of buses

Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00, as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

2.3. The Load Flow Problem and Power Flow Equations

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \tag{2}$$

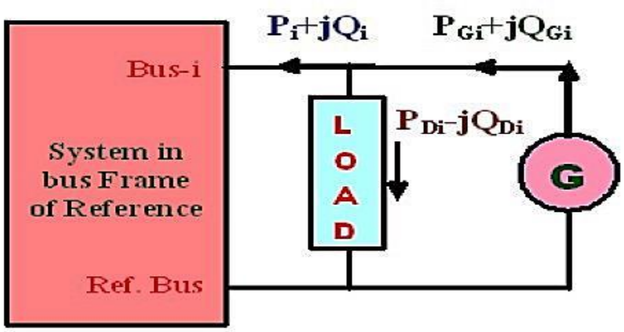


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence, Si = _Sij " i = 1, 2,............. n (3)

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ………..n (4)

where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS (5)

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\, I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\, V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $$V_i \underline{\underline{\Delta}} |V_i| \angle\, \delta_i = |V_i| (\cos\ \delta_i + j \sin\ \delta_i)$$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,(\cos\delta_{ij} + j\sin\delta_{ij})\,(G_{ij} - jB_{ij}) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $$Y_{ij} = |Y_{ij}|\angle\theta_{ij} \qquad \textbf{(11)}$$

Again, we get from (7),

$$S_i = |V_i|\angle\delta_i \sum_{j=1}^{n} |Y_{ij}|\angle-\theta_{ij}\,|V_j|\angle-\delta_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos(-\theta_{ij} + \delta_i - \delta_j)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\, i = 1, 2, \ldots\ldots\ldots n, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\sin-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\, i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the 'power flow equations' or the 'load flow equations' in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

2.4. Data for Load Flow

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

1. **System data**: It includes:
    - Number of buses-*n*,
    - Number of PV buses,
    - Number of loads,
    - Number of transmission lines,
    - Number of transformers,
    - Number of shunt elements,
    - The slack bus number,
    - Voltage magnitude of slack bus (angle is generally taken as 0o),
    - Tolerance limit,
    - Base MVA and
    - Maximum permissible number of iterations.
2. **Generator bus data**: For every PV bus *i*, the data required includes the
    - Bus number,
    - Active power generation P_{Gi},
    - The specified voltage magnitude
    - Minimum reactive power limit Qi,min, and
    - Maximum reactive power limit Qi,max.
3. **Load data**: For all loads the data required includes the
    - Bus number,
    - Active power demand P_{Di}, and
    - The reactive power demand Q_{Di}.
4. **Transmission line data**: For every transmission line connected between buses *i* and *k* the data includes the

- Starting bus number *i*,
- Ending bus number *k*,
- *R*esistance of the line,
- Reactance of the line and
- Half line charging admittance.

5. **Transformer data:** For every transformer connected between buses *i* and *k* the data to be given includes:

- Starting bus number *i*,
- Ending bus number *k*,
- Resistance of the transformer,
- Reactance of the transformer, and
- The off nominal turns-ratio *a*.

6. **Shunt element data**: The data needed for the shunt element includes:

- The bus number where element is connected, and
- The shunt admittance (Gsh + j Bsh).

2.5. Gauss – Seidel (GS) Method

The GS method is an iterative algorithm for solving nonlinear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from Equation (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \quad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \quad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \quad \forall\, i = 2, 3, \ldots\ldots\ldots.n \quad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

2.5.1. Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be 1.0□0^0. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any $(k+1)st$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \forall \text{ i=2,3,…n} \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall \text{ i = 2,3,…n} \qquad \textbf{(19)}$$

Where, ε is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

5. Compute all line flows.
6. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\operatorname{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij} V_j\right\}$$

Where *Im* stands for the imaginary part. At any *(k+1)*st iteration, at the PV bus-*i,*

$$Q_i^{(k+1)} = -\operatorname{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij} V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \tag{22}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $(k+1)$ i Q computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$ If $Q_i > Q_{i,max}$

Then $Q_i = Q_{i,min}$. Then $Q_i = Q_{i,max}$.

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \tag{24}$$

where α is a real number. When α =1, the value of $(k+1)$ is the computed value. If 1<α<2 then the value computed is extrapolated. Generally α is taken between 1.6 to 2.0, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

2.6. Examples on GS load flow analysis

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

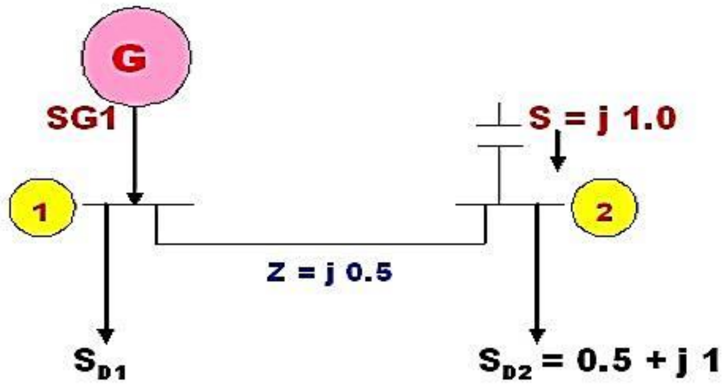


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776\ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\,0.23529 = 0.970145\ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\,0.249999 \;= 0.970261\;\angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\,0.248963 = 0.966237\;\angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\,0.25 \;\; = 0.966237\;\angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10^{-4} pu, the iterations can be stopped.

To compute line flow,

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 \;-\; 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472\;\angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472\;\angle\;14.931^0$$

$$= 0.5 + j\,0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

$$= 0.517472\;\angle -194.93^0$$

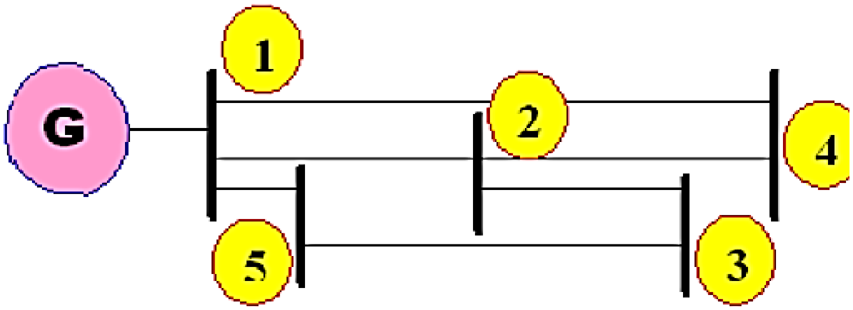
$$S_{21} = V_2 I_{21}^* = -\,0.5 + j\,0.0 \text{ pu}$$

The total loss in the line is given by

$$S_{12} + S_{21} = j\;0.133329\;pu$$

Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2: For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP}\rvert$ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $\quad P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be $V_1^0 = 1.02+j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^0\}\right]$$

$$= 0.98140\angle -3.0665^o = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore$ $\delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o$ ($\delta_{ik} = \delta_i - \delta_k$); $\quad \delta_{32} = 3.0665^o$

$Q_3 = 1.04$ [1.02 (0.0+j0.0) + 0.9814 {−1.17647 × sin(3.0665°) – 4.70588 ×cos(3.0665°)}+1.04{–9.41176 ×cos(0°)}+1.0 {0.0 + j0.0}+1.0{–4.70588×cos(0°)}]

$= 1.04\,[-4.6735 + 9.78823 - 4.70588] = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588)\times(0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times(1\angle 0^o)\}\right]$$

$$= 1.05569\angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^0$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294)\times(0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle -1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\qquad V_2 = 0.98140 \angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\qquad V_4 = 0.955715\angle -7.303^0$ pu

and $\qquad V_5 = 0.994618\angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses
(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu
(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and $0.25 \le Q2 \le 1.0$ pu.

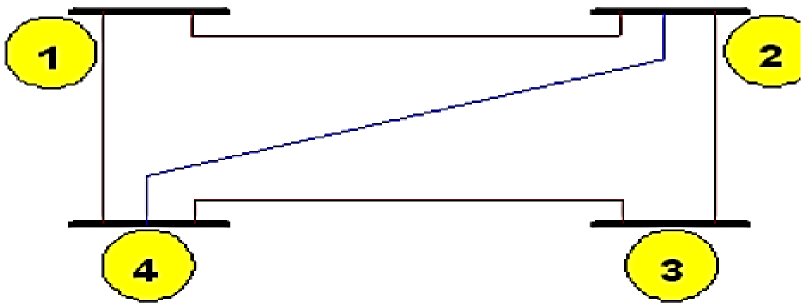


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\ -2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\ 0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

<u>Case(i)</u>: All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0\angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014\angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108\angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467\angle -0.51^o$$

Hence

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.02014\ \angle 2.605^0$ pu

$V_3^1 = 1.03108\ \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467\ \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\big[|V_1|(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2|(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22}) + |V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + |V_4|(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24})\big]$$

$$= 1.04\ [1.04\ \{-6.0\} + 1.04\ \{11.0\} + 1.0\{-2.0\} + 1.0\ \{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\} - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\ \angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\} - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.035587\angle -4.951^o \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$

$$= 0.9985\angle -0.178^o$$

Hence at end of 1st iteration we have:

$$V_1^1 = 1.04\ \angle 0^0 \text{ pu} \qquad V_2^1 = 1.04\ \angle 1.846^0 \text{ pu}$$

$$V_3^1 = 1.035587\ \angle -4.951^0 \text{ pu} \qquad V_4^1 = 0.9985\angle -0.178^0 \text{ pu}$$

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04\ \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645\ \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546\ \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446\angle 4.896^0 \text{ pu}$$

2.7. Limitations of GS Load Flow Analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

Power System Analysis-2 (18EE71)
2021-22

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Module 3

Load Flow Studies (Continued)

Table of Contents

3.1. Newton-Raphson Method

Although the Gauss-Seidel was the first popular method for load flow calculations, the Newton-Raphson method is now commonly used. The Newton-Raphson (NR) method has better convergence characteristics and for many systems is faster than the Gauss-Seidel method; the former has a much larger time per iteration but requires very few iterations (four is general), whereas the Gauss-Siedel requires up to 30 iterations, the number increasing with the size of system.

NR method is used to solve a system of non-linear algebraic equations of the form f(x)=0. Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \tag{25}$$

Let $x_1^0, x_2^0 \ldots\ldots\ldots\ldots x_n^0$, be the initial guess of unknown variables and $\Delta x_1^0, \Delta x_2^0 \ldots\ldots \Delta x_n^0$ be the respective corrections. Therefore,

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0 \ldots\ldots\ldots x_n^0 + \Delta x_n^0) = 0 \qquad i = 1,2\ldots\ldots n \tag{26}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0 \ldots\ldots x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)\Delta x_2^0 + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right)\Delta x_n^0\right]$$

$$+ \text{Higher order terms} = 0 \qquad \forall\ i = 1,2\ldots\ldots n \tag{27}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at $(x_1^0, x_2^0 \ldots\ldots\ldots\ldots x_0^n)$. If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \cdot \\ \cdot \\ \cdot \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^0 \end{bmatrix} = 0 \tag{28}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \quad \textbf{(29)}$$

And $$X^1 = X^0 + \Delta X^0 \quad \textbf{(30)}$$

Here, the matrix [J] is called the Jacobian matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32 are of the form F(x) = 0. Thus NR method can be applied to solve them. Equation: (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respec to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partia derivative of P_{cal}, since P_{sp} is a constant. The various computations involved ar discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \qquad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \qquad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available fron previous iteration. The elements of the Jacobian are found using the above equation: as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin \delta_{ik}) + B_{ik} \cos \delta_{ik}\}$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1\\k\neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}H & N\\ M & L\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \dfrac{\Delta|V|}{|V|}\end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

3.2. Decoupled Load Flow

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected systems, alternative solution methods are possible, taking into accounbt certain observations are made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\dfrac{\partial Q_i}{\partial |V_j|}$ is much larger than $\dfrac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\dfrac{\partial P_i}{\partial \delta_j}$ is much larger than $\dfrac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \dfrac{\Delta |V|}{|V|} \end{bmatrix} \tag{37}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta \delta^{(r)} \\ \dfrac{\Delta |V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta |V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

3.3. Fast Decoupled Load Flow

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)
- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = [|V_i||V_j|B'_{ij}][\Delta\delta]$$

$$[\Delta Q] = [|V_i||V_j|B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \tag{38}$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \tag{39}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

3.4. Comparison of Load Flow Methods

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

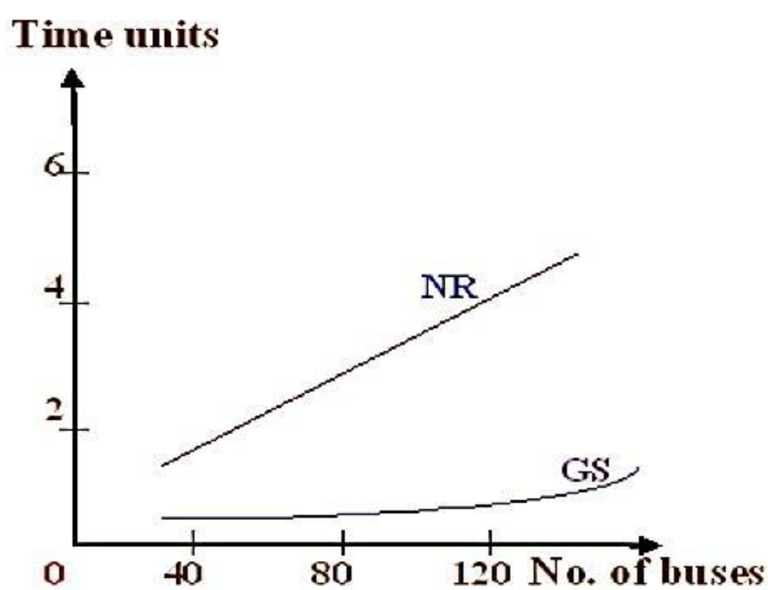

Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution.

Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

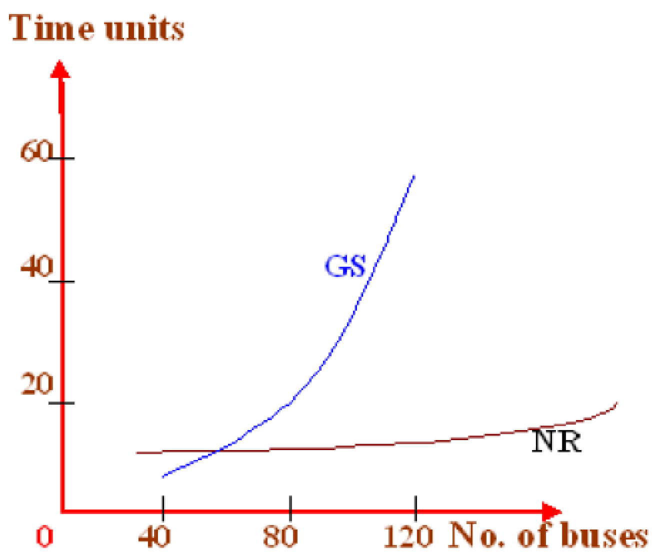

Figure 5. Total time of Iteration in GS and NR methods

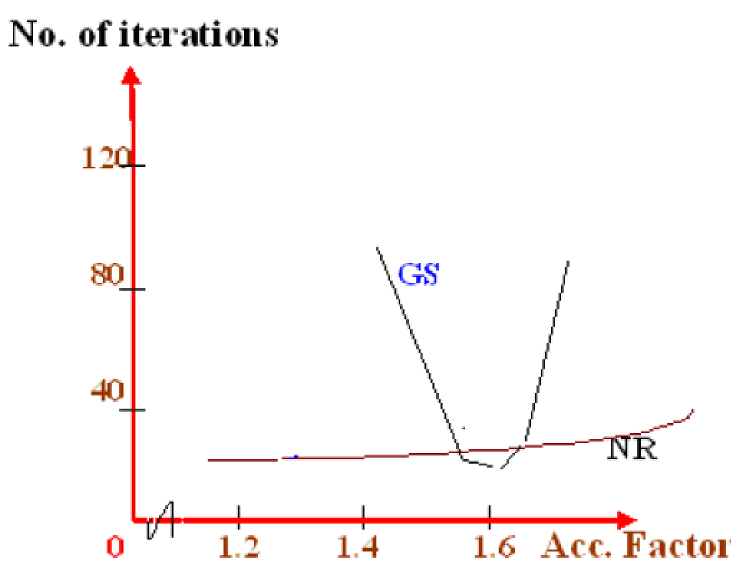

Figure 6. Influence of acceleration factor on load flow methods

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade-off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.

Power System Analysis-2 (18EE71)
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Module 4

Economic Operation of Power System & Unit Commitment

Table of Contents

4.1. Introduction

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants. The operation economics can again be subdivided into two parts.

i) ***Problem of economic dispatch***, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.

ii) ***Problem of optimal power flow***, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

During operation of the plant, a generator may be in one of the following states:

1. Base supply without regulation: the output is a constant.
2. Base supply with regulation: output power is regulated based on system load.
3. Automatic non-economic regulation: output level changes around a base setting as area control error changes.
4. Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

4.2. Performance Curves

i) Input-Output Curve

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) perhour versus the power output of the plant in MW as shown in Fig1.

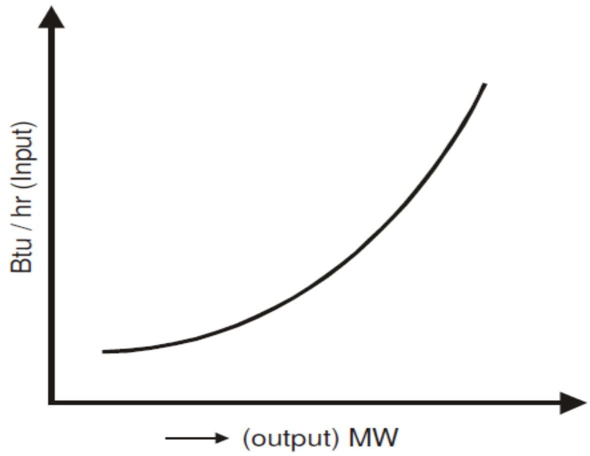


Fig 1: Input – output curve

ii) Heat Rate Curve

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel –efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .2

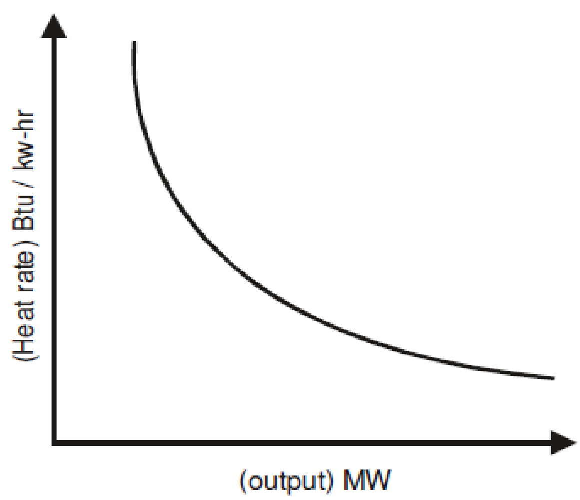


Fig 2: Heat rate curve.

iii) Incremental Fuel Rate Curve

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

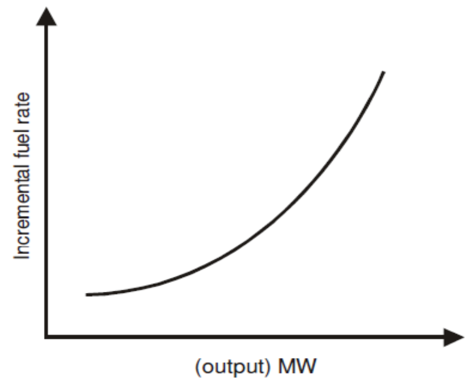


Fig 3: Incremental fuel rate curve

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

iv) Incremental cost curve

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $ / Btu). The curve in shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

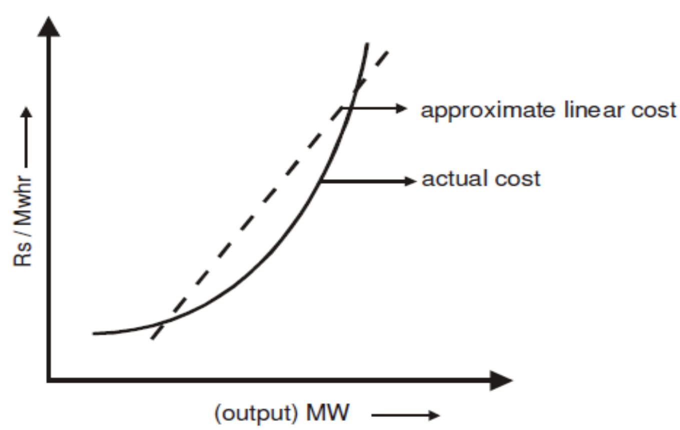


Fig 4: Incremental cost curve

In general, the fuel cost F_i for a plant, is approximated as a quadratic function of the generated output P_{Gi}.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \ \ Rs / h$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \text{Rs / MWh}$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between P_{Gmin}, the minimum loading limit, below which it is technically infeasible to operate a unit and P_{Gmax}, which is the maximum output limit.

4.3. Economic Generation Scheduling Neglecting Losses And Generator Limits

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand P_D. Consider a system with $\mathbf{n}_g$ number of generating plants supplying the total demand P_D. If Fi is the cost of plant i in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

where F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

This is a constrained optimization problem, which can be solved by Lagrange's method.

Lagrange Method for Solution of Economic Schedule

The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$£ = F_T + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial \lambda} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial £}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant Fi depends only on its own output P_{Gi}, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \;; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that l is dependent on the demand and the coefficients of the cost function.

Example 1.

The fuel costs of two units are given by

$F_1 = 1.5 + 20\ P_{G1} + 0.1\ P_{G1}^2$ Rs/h

$F_2 = 1.9 + 30\ P_{G2} + 0.1\ P_{G2}^2$ Rs/h

P_{G1}, P_{G2} are in MW. Find the optimal schedule neglecting losses, when the demand is 200 MW.

Solution:

$$\frac{dF_1}{dP_{G1}} = 20 + 0.2P_{G1} \quad \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 30 + 0.2P_{G2} \quad \text{Rs / MWh}$$

$$P_D = P_{G1} + P_{G2} = 200 \text{ MW}$$

For economic schedule

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \lambda$$

$$20 + 0.2\ P_{G1} = 30 + 0.2\ (200 - P_{G1})$$

Solving we get, $P_{G1} = 125$ MW

$P_{G2} = 75$ MW

$$\lambda = 20 + 0.2\,(125) = 45 \text{ Rs / MWh}$$

Example 2

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\, P_{G1} + 0.004\, P_{G1}^2$

$F_2 = 500 + b_2\, P_{G2} + c_2\, P_{G2}^2$

where P_{G1}, P_{G2} are in MW

(a) The incremental cost of power, λ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients b_2 and c_2.

Solution:

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300 \text{ MW}$$

b) $$P_{G1} = \frac{\lambda - b_1}{2C_1} = \frac{10 - 6}{2 \times 0.004} = 500 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \text{ MW}$$

c) $$P_{G2} = \frac{\lambda - b_2}{2c_2}$$

From (a) $$300 = \frac{8.0 - b_2}{2c_2}$$

From (b) $$800 = \frac{10.0 - b_2}{2c_2}$$

Solving we get $b_2 = 6.8$

$c_2 = 0.002$

4.4. Economic Schedule Including Limits on Generator (Neglecting Losses)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \leq P_{Gi} \leq P_{Gi\,(max)} \,;\, i = 1, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

Example 3: Incremental fuel costs in \$ / MWh for two units are given below:

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 2.0 \ \$ / \text{MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.012P_{G2} + 1.6 \$ / \text{MWh}$$

The limits on the plants are P_{min} = 20 MW, P_{max} = 125 MW. Obtain the optimal schedule if the load varies from 50 – 250 MW.

Solution:

The incremental fuel costs of the two plants are evaluated at their lower limits and upper limits of generation.

At $P_{G\,(min)}$ = 20 MW.

$$\lambda_{1(min)} = \frac{dF_1}{dP_{G1}} = 0.01\text{x } 20 + 2.0 = 2.2\$ / \text{MWh}$$

$$\lambda_{2(min)} = \frac{dF_2}{dP_{G2}} = 0.012 \text{ x } 20 + 1.6 = 1.84 \ \$ / \text{MWh}$$

At $P_{G\,(Max)}$ =125 Mw

$$\lambda_{1(max)} = 0.01 \times 125 + 2.0 = 3.25\ \$ / MWh$$

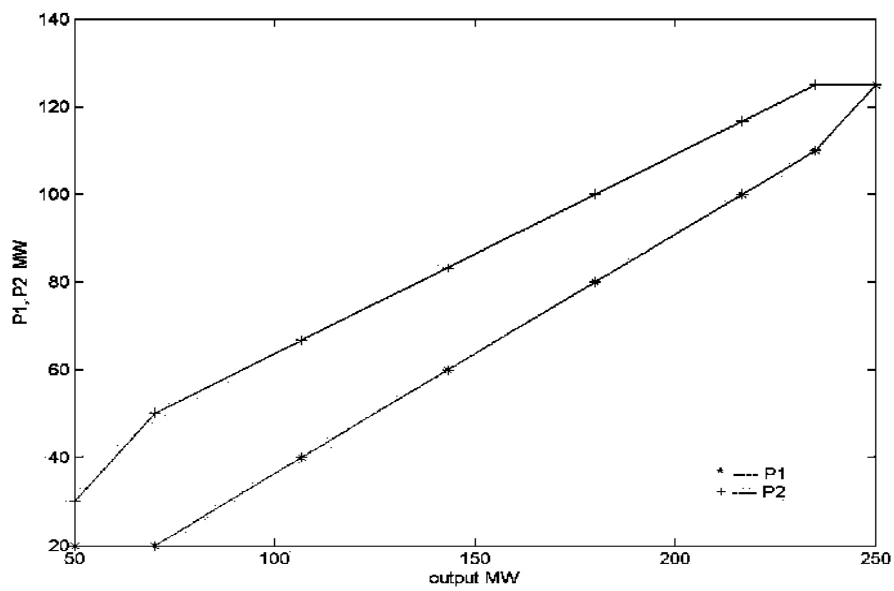
$$\lambda_{2(max)} = 0.012 \times 125 + 1.6 = 3.1\ \$ / MWh$$

Now at light loads unit 1 has a higher incremental cost and hence will operate at its lower limit of 20 MW. Initially, additional load is taken up by unit 2, till such time its incremental fuel cost becomes equal to 2.2$ / MWh at PG2 = 50 MW. Beyond this, the two units are operated with equal incremental fuel costs. The contribution of each unit to meet the demand is obtained by assuming different values of l; When l = 3.1 $ / MWh, unit 2 operates at its upper limit. Further loads are taken up by unit 1. The computations are show in Table below.

Table Plant output and output of the two units

$\frac{dF_1}{dP_{G1}}$ $/MWh	$\frac{dF_2}{dP_{G2}}$ $/MWh	Plant λ $/MWh	P_{G1} MW	P_{G2} MW	Plant Output MW
2.2	1.96	1.96	20⁺	30	50
2.2	2.2	2.2	20⁺	50	70
2.4	2.4	2.4	40	66.7	106.7
2.6	2.6	2.6	60	83.3	143.3
2.8	2.8	2.8	80	100	180
3.0	3.0	3.0	100	116.7	216.7
3.1	3.1	3.1	110	125*	235

For a particular value of l, P_{G1} and P_{G2} are calculated. Fig below Shows plot of each unit output versus the total plant output. For any particular load, the schedule for each unit for economic dispatch can be obtained.



Example 4.

In example 3, what is the saving in fuel cost for the economic schedule compared to the case where the load is shared equally. The load is 180 MW.

Solution:

From Table it is seen that for a load of 180 MW, the economic schedule is PG1 = 80 MW and PG2 = 100 MW. When load is shared equally PG1 = PG2 = 90 MW. Hence, the generation of unit 1 increases from 80 MW to 90 MW and that of unit 2 decreases from 100 MW to 90 MW, when the load is shared equally. There is an increase in cost of unit 1 since PG1 increases and decrease in cost of unit 2 since PG2 decreases.

$$\text{Increase in cost of unit 1} = \int_{80}^{90}\left(\frac{dF_1}{dP_{G1}}\right)dP_{G1}$$

$$= \int_{80}^{90}(0.01P_{G1} + 2.0)dP_{G1} = 28.5\ \$ / \text{h}$$

$$\text{Decrease in cost of unit 2} = \int_{100}^{90}\left(\frac{dF_2}{dP_{G2}}\right)dP_{G2}$$

$$= \int_{100}^{90}(0.012P_{G2} + 1.6)dP_{G2} = -27.4\ \$ / \text{h}$$

Total increase in cost if load is shared equally = 28.5 – 27.4 = 1.1 \$ / h

Hence the saving in fuel cost is 1.1 \$ / h if coordinated economic schedule is used.

4.5. Economic Dispatch Including Transmission Losses

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such That $$\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) = 0\,;\ i = 1\ldots\ldots n_g$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant ***i***, $\mathbf{L_i}$. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i\,;\ i = 1\ldots\ldots.n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered. A rigorous general expression for the loss PL is given by

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn} + \Sigma_n P_{Gn} B_{no} + B_{oo}$$

where Bmn, Bno , Boo called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values. A simpler expression is

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems have shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, Bmn = Bnm and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

Example 5

A generator is supplying a load. An incremental change in load of 4 MW requires generation to be increased by 6 MW. The incremental cost at the plant bus is Rs 30 /MWh. What is the incremental cost at the receiving end?

Solution:

$$\frac{dF_1}{dP_{G1}} = 30$$

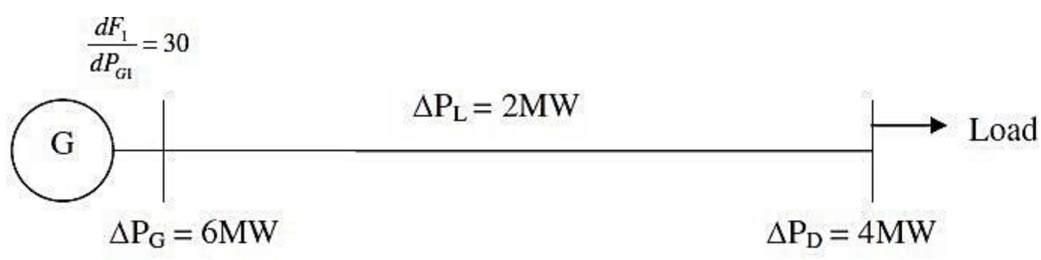


Fig ; One line diagram of example 5

$$\Delta P_L = \Delta P_G - \Delta P_D = 2 \text{ MW}$$

λ at receiving end is given by

$$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{\Delta P_G}{\Delta P_D} = 30 \times \frac{6}{4} = 45 \text{ Rs / MWh}$$

$$\text{or } \lambda = \frac{dF_1}{dP_{G1}} \times \frac{1}{1 - \frac{\Delta P_L}{\Delta P_G}} = 30 \times \frac{1}{1 - \frac{2}{6}} = 45 \text{ Rs / MWh}$$

Example 6

In a system with two plants, the incremental fuel costs are given by

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 20 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \text{ Rs / MWh}$$

The system is running under optimal schedule with $P_{G1} = P_{G2} = 100$ MW.

If $\frac{\partial P_L}{\partial P_{G2}} = 0.2$, find the plant penalty factors and $\frac{\partial P_L}{\partial P_{G1}}$.

Solution:

For economic schedule,

$$\frac{dF_i}{dP_{Gi}} L_i = \lambda ; \qquad L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}};$$

For plant 2, $P_{G2} = 100$ MW

$$\therefore (0.015 \times 100 + 22.5)\frac{1}{1 - 0.2} = \lambda.$$

Solving, $\lambda = 30 \text{ Rs / MWh}$

$$L_2 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{dF_1}{dP_{G1}} L_1 = \lambda \Rightarrow (0.01 \text{x} 100 + 20) L_1 = 30$$

$L_1 = 1.428$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

$$1.428 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} \text{ ; Solving } \frac{\partial P_L}{\partial P_{G1}} = 0.3$$

Example 7

A two bus system is shown in Fig. 8.8 If 100 MW is transmitted from plant 1 to the load, a loss of 10 MW is incurred. System incremental cost is Rs 30 / MWh. Find P_{G1}, P_{G2} and power received by load if

$$\frac{dF_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \ \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \ \text{Rs / MWh}$$

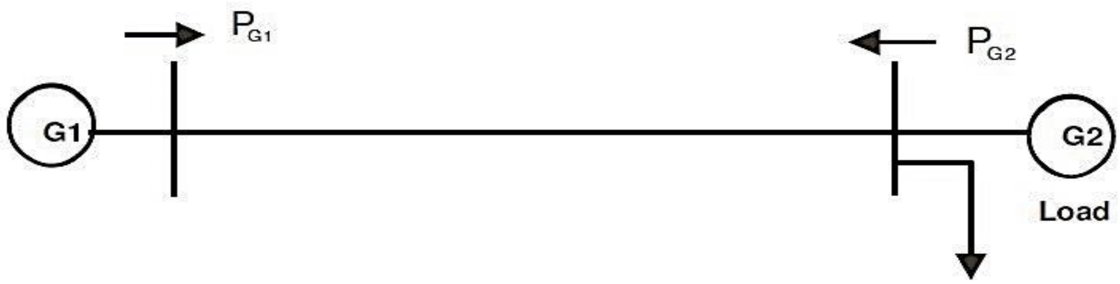


Fig One line diagram of example 7

Solution:

Since the load is connected at bus 2 , no loss is incurred when plant two supplies the load.

Therefore in (8.36) $B_{12} = 0$ and $B_{22} = 0$

$$P_L = B_{11}P_{G1}^2; \qquad \frac{\partial P_L}{\partial P_{G1}} = 2B_{11}P_{G1}; \qquad \frac{\partial P_L}{\partial P_{G2}} = 0.0$$

From data we have $P_L = 10$ MW, if $P_{G1} = 100$ MW

$10 = B_{11}\,(100)^2$

$B_{11} = 0.001 \quad MW^{-1}$

Coordination equation with loss is

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

For plant 1 $\quad \dfrac{dF_1}{dP_{G1}} + \lambda\dfrac{\partial P_L}{\partial P_{G1}} = \lambda$

$(0.02\ P_{G1} + 16.0) + 30\ (2 \times 0.001 \times P_{G1}) = 30$

$0.08\ P_{G1} = 30 - 16.0$. From which, $P_{G1} = 175$ MW

For Plant 2 $\dfrac{dF_2}{dP_{G1}} + \lambda \dfrac{\partial P_L}{\partial P_{G2}} = \lambda$

$0.04\ P_{G2} + 20.0 = 30$ or $P_{G2} = 250$ MW

Loss $= B_{11}\ P_{G1}^2 = 0.001 \times (175)^2 = 30.625$ MW

$P_D = (P_{G1} + P_{G2}) - P_L = 394.375$ MW

4.6. Derivation of Transmission Loss Formula

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference

(ii) The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig below.

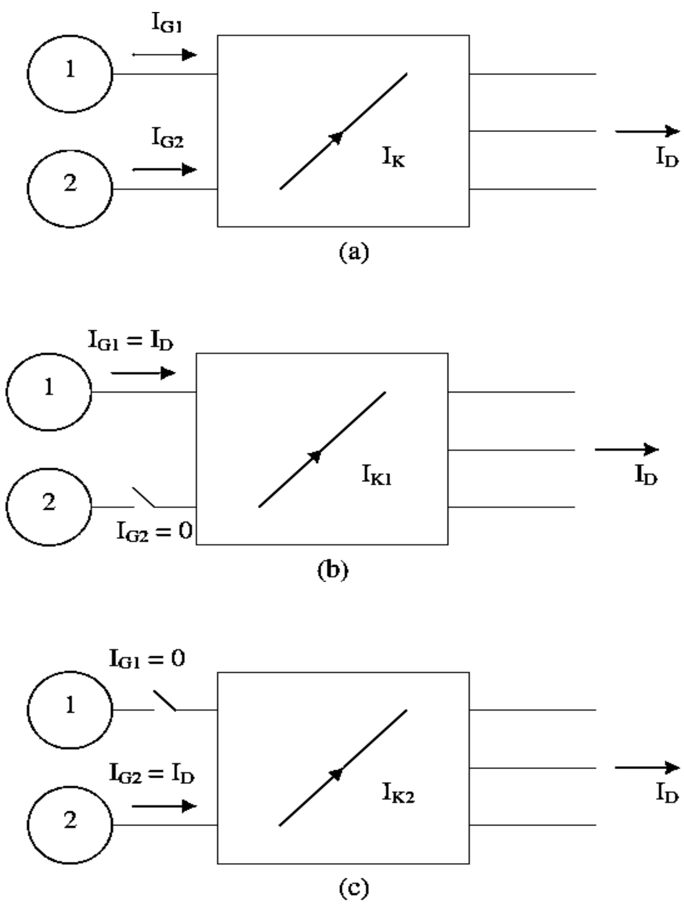


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1}\, I_{G1} + N_{K2}\, I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2 .$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{array}{l} N_{K1}{}^2|I_{G1}|^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}{}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{array}$$

$$= N_{K1}{}^2|I_{G1}|^2 + N_{K2}{}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^{\ 2}}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^{\ 2}R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}^{\ 2}}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^{\ 2}R_K$$

$$P_L = P_{G1}^{\ 2}B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^{\ 2}B_{22}$$

where
$$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^{\ 2}R_K$$

$$B_{12} = \frac{\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K$$

$$B_{22} = \frac{1}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^{\ 2}R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^{\ 2}}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^{\ 2} + \ldots\ldots + \frac{P_{Gn}^{\ 2}}{|V_n|^2(\cos\phi_n)^2}\sum_K N_{Kn}^{\ 2}R_K + 2\sum_{\substack{p,q=1\\ p\neq q}}^{n}\frac{P_{GP}P_{Gq}\cos(\sigma_p-\sigma_q)}{|V_p||V_q|\cos\phi_p\cos\phi_q}\sum_k N_{KP}N_{Kq}R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n}\sum_{q=1}^{n} P_{Gp}B_{Pq}P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p-\sigma_q)}{|V_p||V_q|\cos\phi_P\cos\phi_q}\sum_K N_{KP}N_{Kq}R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Example 8

Calculate the loss coefficients in pu and MW^{-1} on a base of 50MVA for the network of Fig below. Corresponding data is given below.

$I_a = 1.2 - j\,0.4$ pu $\qquad Z_a = 0.02 + j\,0.08$ pu

$I_b = 0.4 - j\,0.2$ pu $\qquad Z_b = 0.08 + j\,0.32$ pu

$I_c = 0.8 - j\,0.1$ pu $\qquad Z_c = 0.02 + j\,0.08$ pu

$I_d = 0.8 - j\,0.2$ pu $\qquad Z_d = 0.03 + j\,0.12$ pu

$I_e = 1.2 - j\,0.3$ pu $\qquad Z_e = 0.03 + j\,0.12$ pu

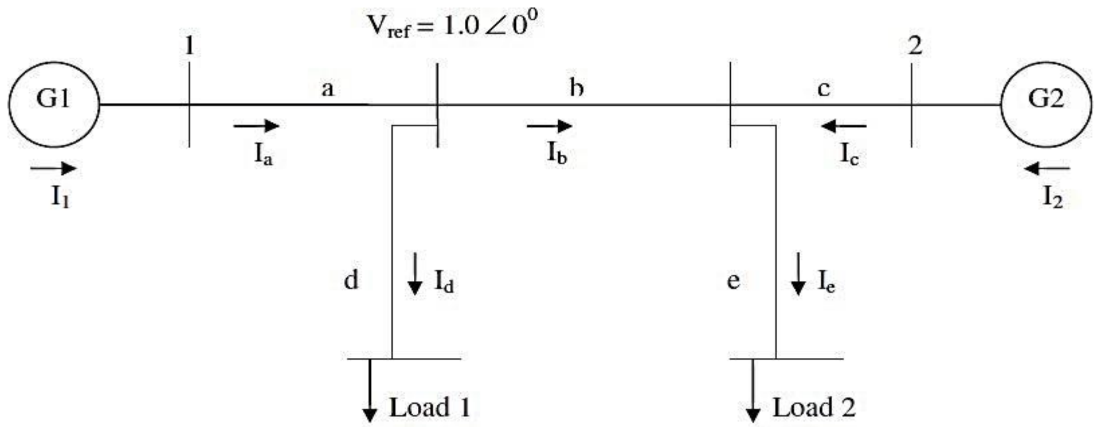


Fig : Example 8

Solution:

Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061\ \angle -14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246\ \angle -14.03^0\ A$

$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

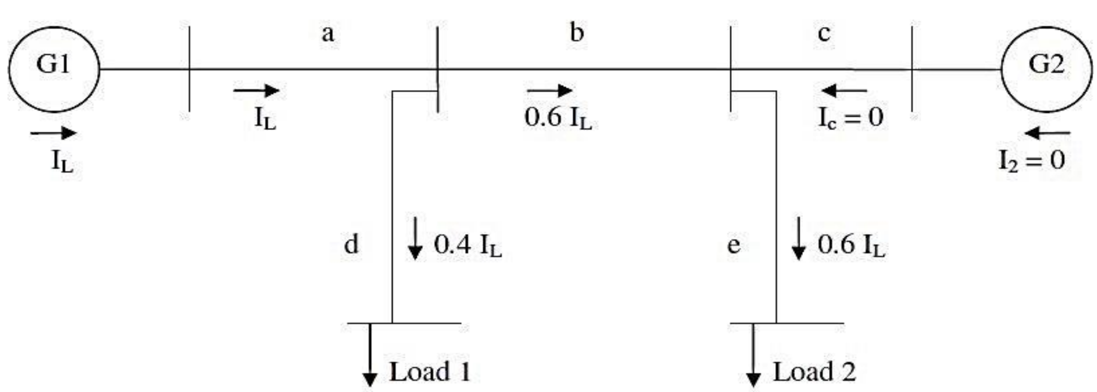


Fig a : Generator 1 supplying the total load

$$N_{a1} = \frac{I_a}{I_L} = 1.0;\ N_{b1} = \frac{I_b}{I_L} = 0.6;\ N_{C1} = 0;\ N_{d1} = 0.4;\ N_{e1} = 0.6.$$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

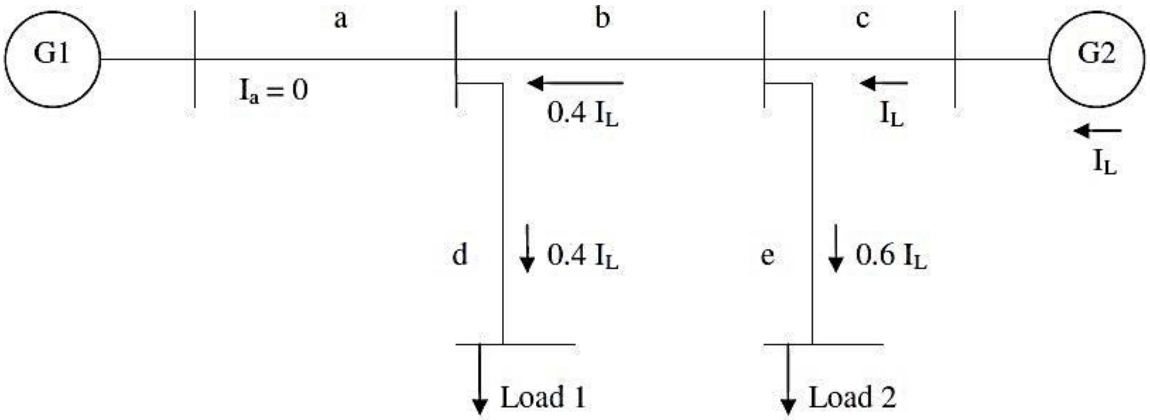


Fig b: Generator 2 supplying the total load

$N_{a2} = 0$; $N_{b2} = -0.4$; $N_{c2} = 1.0$; $N_{d2} = 0.4$; $N_{e2} = 0.6$

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$$= 1\angle 0^0 + (1.2 - j\,0.4)\,(0.02 + j0.08)$$

$$= 1.06\angle 4.78^0 = 1.056 + j\,0.088 \text{ pu.}$$

$$V_2 = V_{ref} - I_b Z_b + I_c Z_c$$

$$= 1.0\ \angle 0^0 - (0.4 - j\,0.2)\,(0.08 + j\,0.32) + (0.8 - j\,0.1)\,(0.02 + j\,0.08)$$

$$= 0.928 - j\,0.05 = 0.93\ \angle -3.10^0 \text{ pu.}$$

Current Phase angles

$$\sigma_1 = \text{angle of } I_1 (= I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\sigma_2 = \text{angle of } I_2 (= I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$$\phi_1 = 4.78^0 + 18.43 = 23.21^0; \cos\phi_1 = 0.92$$

$$\phi_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\phi_2 = 0.998$$

$$B_{11} = \frac{\sum_K N_{K1}{}^2 R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$$B_{11} = \frac{\sum_K N_{K1}{}^2 R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

= 0.0677 pu

$= 0.0677 \text{ x } \frac{1}{50} = 0.1354 \times 10^{-2}\,\text{MW}^{-1}$

$$B_{12} = \frac{Cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} \sum_K N_{K1} N_{K2} R_K$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)} [-0.4 \times 0.6 \times 0.08 + 0.4 \times 0.4 \times 0.03 + 0.6 \times 0.6 \times 0.03]$$

= - 0.00389 pu

= - 0.0078 x 10^{-2} MW^{-1}

$$B_{22} = \frac{\sum_K N_{K2}{}^2 R_K}{|V_2|^2 (\cos\phi_2{}^2)}$$

$$= \frac{(-0.4)^2 0.08 + 1.0^2 \times 0.02 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(0.93)^2 (0.998)^2}$$

$= 0.056\text{pu} = 0.112 \times 10^{-2}\,\text{MW}^{-1}$

Unit Commitment

4.7. Introduction

The total load of the power system is not constant but varies throughout the day and reaches a different peak value from one day to another. It follows a particular hourly load cycle over a day. There will be different discrete load levels at each period as shown in Figure 4.1 below.

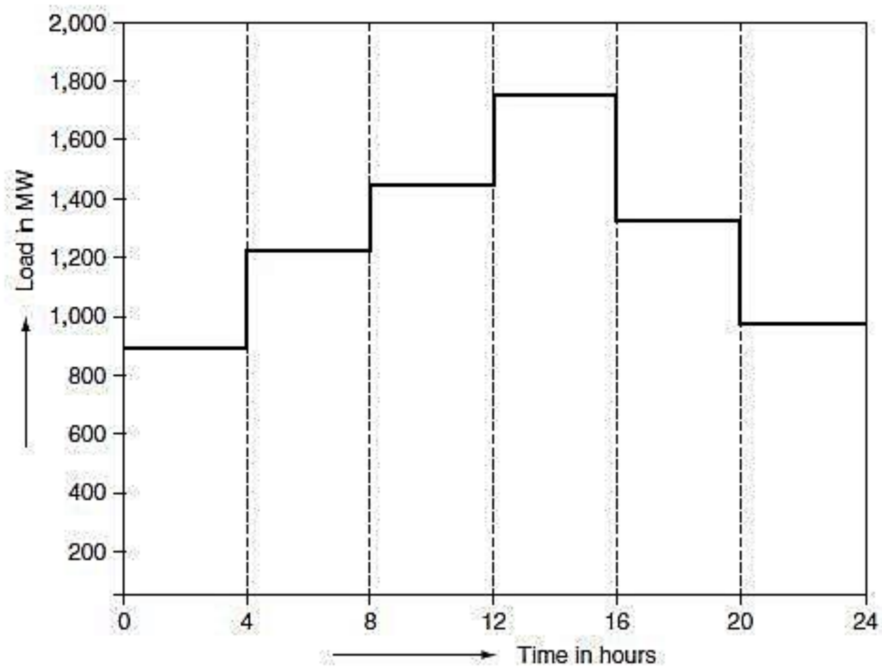


Fig 4.1: Discrete levels of system load of daily load cycle

Due to the above reason, it is not advisable to run all available units all the time, and it is necessary to decide in advance which generators are to startup, when to connect them to the network, the sequence in which the operating units should be shut down, and for how long. The computational procedure for making such decisions is called unit commitment (UC), and a unit when scheduled for connection to the system is said to be committed.

The problem of UC is nothing but to determine the units that should operate for a particular load. To 'commit' a generating unit is to 'turn it on', i.e., to bring it up to speed, synchronize it to the system, and connect it, so that it can deliver power to the network.

4.8. Constraints in Unit Commitment

There are many constraints to be considered in solving the UC problem.

4.8.1. Spinning reserve

It is the term used to describe the total amount of generation available from all synchronized units on the system minus the present load and losses being supplied. Here, the synchronized units on the system may be named units spinning on the system.

$$\text{Spinning reserve} = \begin{bmatrix}\text{Total generation output of all} \\ \text{synchronized units at a} \\ \text{particular time}\end{bmatrix} - \begin{bmatrix}\text{Load at that time} + \\ \text{Losses at that time}\end{bmatrix}$$

Let P_{Gsp} be the spinning reserve, $P_{G_{i_s}}$ the power generation of the i th synchronized unit, P_D the total load on the system, and p_L the total loss of the system:

$$\therefore\ P_{G_{sp}} = \sum_{i=1}^{n} P_{G_{i_s}} - (P_D + P_L)$$

The spinning reserve must be maintained so that the failure of one or more units does not cause too far a drop in system frequency. Simply, if one unit fails, there must be an ample reserve on the other units to make up for the loss in a specified time period.

The spinning reserve must be a given a percentage of forecasted peak load demand, or it must be capable of taking up the loss of the most heavily loaded unit in a given period of time. It can also be calculated as a function of the probability of not having sufficient generation to meet the load.

The reserves must be properly allocated among fast-responding units and slow-responding units such that this allows the automatic generation control system to restore frequency and quickly interchange the time of outage of a generating unit.

Beyond the spinning reserve, the UC problem may consider various classes of ‘scheduled reserves’ or off-line reserves. These include quick-start diesel or gas-turbine units as well as most hydro-units and pumped storage hydro-units that can be brought on-line, synchronized, and brought upto maximum capacity quickly. As such, these units can be counted in the overall reserve assessment as long as their time to come up to maximum capacity is taken into consideration.

Reserves should be spread well around the entire power system to avoid transmission system limitations (often called ‘bottling’ of reserves) and to allow different parts of the system to run as ‘islands’, should they become electrically disconnected.

4.8.2. Thermal unit constraints

A thermal unit can undergo only gradual temperature changes and this translates into a time period (of some hours) required to bring the unit on the line. Due to such limitations in the operation of a thermal plant, the following constraints are to be considered.

Minimum up-time: During the minimum up-time, once the unit is operating (up state), it should not be turned off immediately.

Minimum down-time: The minimum down-time is the minimum time during which the unit is in ‘down’ state, i.e., once the unit is decommitted, there is a minimum time before it can be recommitted.

Crew constraints: If a plant consists of two or more units, they cannot both be turned on at the same time since there are not enough crew members to attend to both units while starting up.

***Start-up cost*:** In addition to the above constraints, because the temperature and the pressure of the thermal unit must be moved slowly, a certain amount of energy must be expended to bring the unit on-line and is brought into the UC problem as a start-up cost.

The start-up cost may vary from a maximum ‘cold-start’ value to a very small value if the unit was only turned off recently, and it is still relatively close to the operating temperature.

Two approaches to treating a thermal unit during its ‘down’ state:

The first approach (cooling) allows the unit’s boiler to cool down and then heat back up to a operating temperature in time for a scheduled turn-on.

The second approach (banking) requires that sufficient energy be input to the boiler to just maintain the operating temperature.

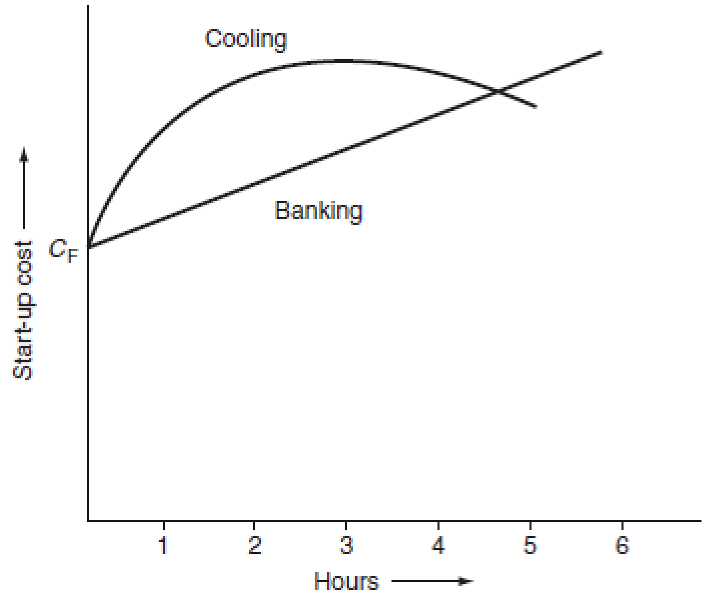


Fig. 4.2: Time-dependent start-up costs

The best approach can be chosen by comparing the costs for the above two approaches.

Let *C*C be the cold-start cost (MBtu), *C* the fuel cost, *C*F the fixed cost (includes crew expenses and maintainable expenses), α the thermal time constant for the unit, *Ct* the cost of maintaining unit at operating temperature (MBtu/hr), and *t* the time the unit was cooled (hr).

Start-up cost when cooling = $C_c\,(1 - e^{-t/\alpha})\,C + C_F$;

Start-up cost when banking $= C_t \times t \times C + C_F$.

Upto a certain number of hours, the cost of banking < cost of cooling is shown in Fig. 4.2.

The capacity limits of thermal units may change frequently due to maintenance or unscheduled outages of various equipment in the plant and this must also be taken into consideration in the UC problem.

4.8.3. Hydro-constraints

As pointed out already that the UC problem is of much importance for the scheduling of thermal units, it is not the meaning of UC that cannot be completely separated from the scheduling of a hydro-unit. The hydro-thermal scheduling will be explained as separated from the UC problem. Operation of a system having both hydro and thermal plants is, however, far more complex as hydro-plants have negligible operation costs, but are required to operate under constraints of water available for hydro-generation in a given period of time.

The problem of minimizing the operating cost of a hydro-thermal system can be viewed as one of minimizing the fuel cost of thermal plants under the constraint of water availability for hydro-generation over a given period of operation.

4.8.4. Must run

It is necessary to give a must-run reorganization to some units of the plant during certain events of the year, by which we yield the voltage support on the transmission network or for such purpose as supply of steam for uses outside the steam plant itself.

4.8.5. Fuel constraints

A system in which some units have limited fuel or else have constraints that require them to burn a specified amount of fuel in a given time presents a most challenging UC problem.

4.9. Unit Commitment—Solution Methods

The most important techniques for the solution of a UC problem are:

i. Priority-list method.
ii. Dynamic programming (DP) method.
iii. Lagrange's relaxation (LR) method.

Now, the priority-list method and the DP method are discussed here.

4.9.1. Priority list method

It is the simplest unit commitment solution which consists of creating a priority list of units.
Full load average production cost= Net heat rate at full load X Fuel

Cost Assumptions:

1. No load cost is zero
2. Unit input-output characteristics are linear between zero output and full load
3. Startup costs are a fixed amount
4. Ignore minimum up time and minimum down time

Steps to be followed

1. Determine the full load average production cost for each units
2. Form priority order based on average production cost
3. Commit number of units corresponding to the priority order
4. Calculate P_{G1}, P_{G2} ,P_{GN} from economic dispatch problem for the feasible combinations only.
5. For the load curve shown, Assume load is dropping or decreasing, determine whether dropping the next unit will supply generation & spinning reserve. If not, continue as it is If yes, go to the next step.
6. Determine the number of hours H, before the unit will be needed again.
7. Check H< minimum shut down time. If not, go to the last step If yes, go to the next step.
8. Calculate two costs, one is the Sum of hourly production for the next H hours with the unit up and second one is the Recalculate the same for the unit down + startup cost for either cooling or banking.
9. Repeat the procedure until the priority.

Merits:

1. No need to go for N combinations
2. Take only one constraint
3. Ignore the minimum up time & down time
4. Complication reduced

Demerits:

1. Startup cost are fixed amount
2. No load costs are not considered

4.9.2. Dynamic-Programming Solution:

Dynamic programming has many advantages over the enumeration scheme, the chief advantage being a reduction in the dimensionality of the problem. Suppose we have found units in a system and any combination of them could serve the (single) load.

There would be a maximum of 24 - 1 = 23 combinations to test. However, if a strict priority order is imposed, there are only four combinations to try:

- Priority 1 unit
- Priority 1 unit + Priority 2 unit
- Priority 1 unit + Priority 2 unit + Priority 3 unit
- Priority 1 unit + Priority 2 unit + Priority 3 unit + Priority 4 unit

The imposition of a priority list arranged in order of the full-load average cost rate would result in a theoretically correct dispatch and commitment only if:

1. No load costs are zero.
2. Unit input-output characteristics are linear between zero output and full load.
3. There are no other restrictions.
4. Start-up costs are a fixed amount.

In the dynamic-programming approach that follows, we assume that:

1. A state consists of an array of units with specified units operating and
2. The start-up cost of a unit is independent of the time it has been off-line
3. There are no costs for shutting down a unit.
4. There is a strict priority order, and in each interval a specified minimum the rest off-line. (i.e., it is a fixed amount).amount of capacity must be operating.

A feasible state is one in which the committed units can supply the required load and that meets the minimum amount of capacity each period.

Forward DP Approach:

One could set up a dynamic-programming algorithm to run backward in time starting from the final hour to be studied, back to the initial hour. Conversely, one could set up the algorithm to run forward in time from the initial hour to the final hour. The forward approach has distinct advantages in solving generator unit commitment. For example, if the start-up cost of a unit is a function of the time it has been off-line (i.e., its temperature), then a forward dynamic-program approach is more suitable since the previous history of the unit can be computed at each stage. There are other practical reasons for going forward. The initial conditions are easily specified and the computations can go forward in time as long as required. A forward dynamic-programming algorithm is shown by the flowchart given in fig 4.4. The recursive algorithm to compute the minimum cost in hour K with combination I is

$F_{cost}(K,I)= \min[P_{cost}(K,I)+S_{cost}(K-1,L:K,I)+F_{cost}(K-1,L)]$

$F_{cost}(K, I)$ = least total cost to arrive at state (K , I)

$P_{cost}(KI,)$ = production cost for state(K ,I)

$S_{cost}(K - 1, L: K , I)$= transition cost from state (K - 1, L) to state(K , I)

State (K, 1) is the Zth combination in hour K. For the forward dynamic programming approach, we define a strategy as the transition, or path, from one state at a given hour to a state at the next hour.

Note that two new variables, X and N, have been introduced

X = number of states to search each period

N = number of strategies, or paths, to save at each step.

These variables allow control of the computational effort (see below Figure 4.3). For n complete enumeration, the maximum number of the value of X or N is $2^n – 1$.

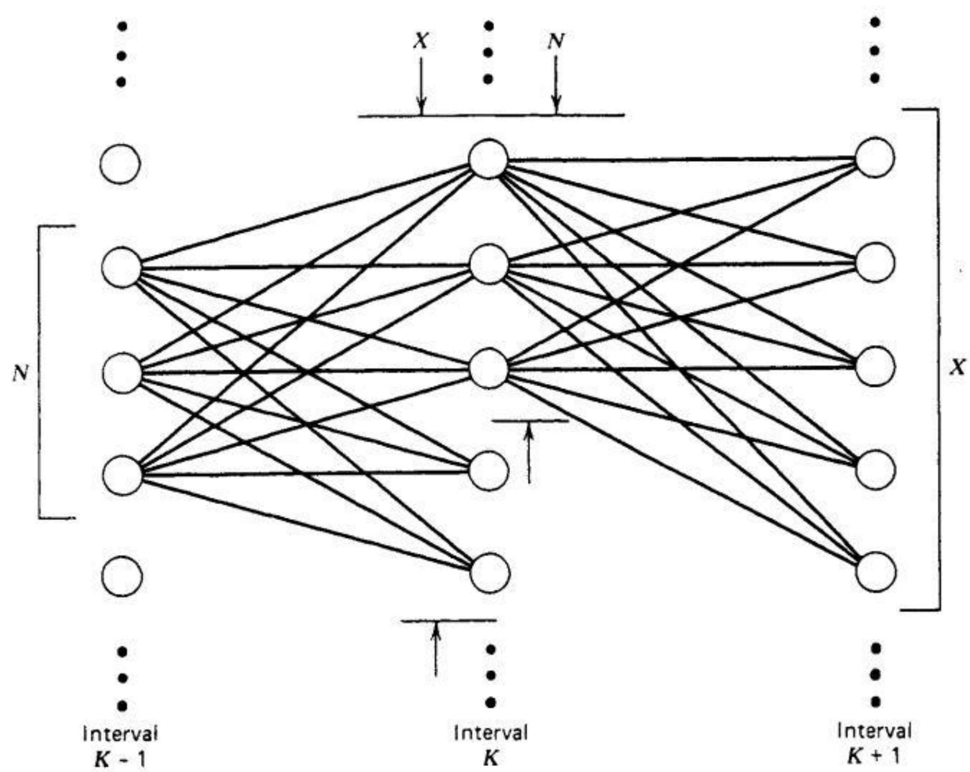


Fig 4.3: Restricted search paths in DP algorithm with N = 3 and X = 5

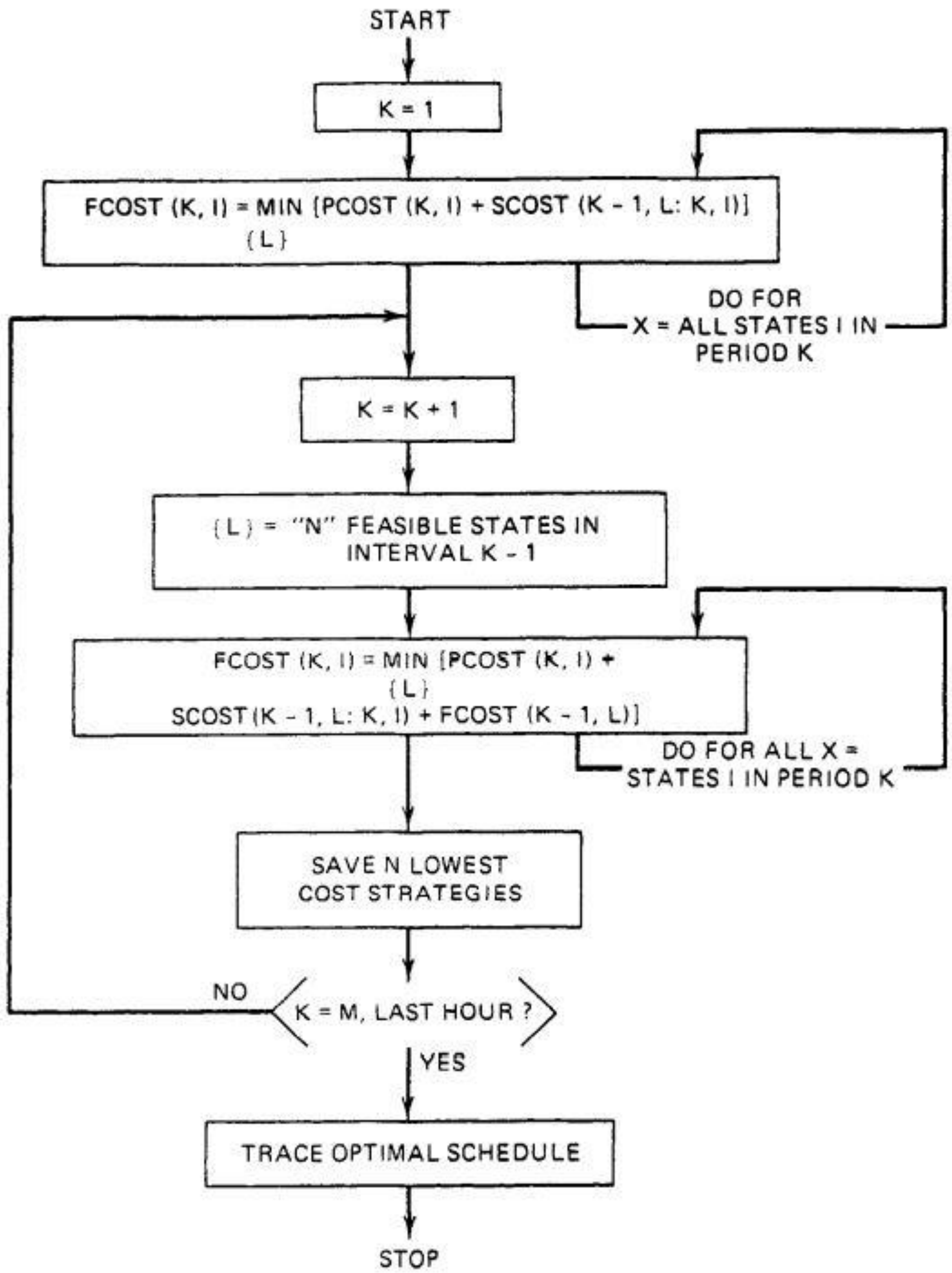


Fig 4.4: Flow chart of Unit commitment via forward dynamic programming

Power System Analysis-2 (18EE71)

2021-22

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Module 5

Symmetrical Fault Analysis & Power System Stability

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5.1. Z Bus Formulation by Step by Step Building Algorithm

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\overline{I}_{bus}$$

When expanded so as to refer to a n bus system, (9) will be of the form

$$\begin{aligned}
E_1 &= Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + .Z_{1k}I_k \ldots + Z_{1n}I_n \\
&\vdots \\
E_k &= Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots + Z_{kn}I_n \\
&\vdots \\
E_n &= Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots + Z_{nk}I_k + \ldots + Z_{nn}I_n
\end{aligned} \qquad (10)$$

Now assume that the bus impedance matrix Zbus is known for a partial network of m buses and a known reference bus. Thus, Zbus of the partial network is of dimension mxm. If now a new element is added between buses p and q we have the following two possibilities:

(i) p is an existing bus in the partial network and q is a new bus; in this case p-q is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both p and q are buses existing in the partial network; in this case p-q is a **link** added to the p-network as shown in Fig 1b.

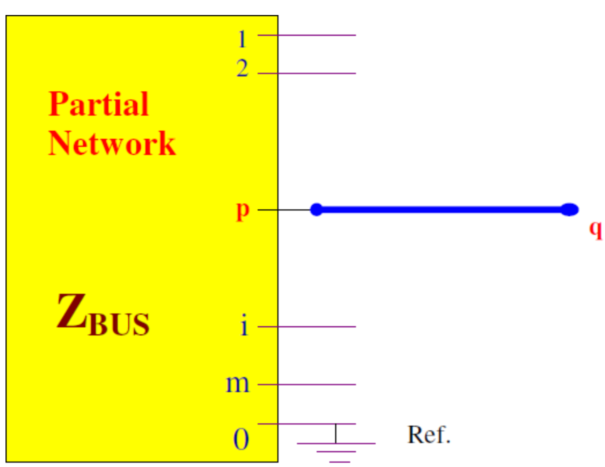


Fig 1a. Addition of branch p-q

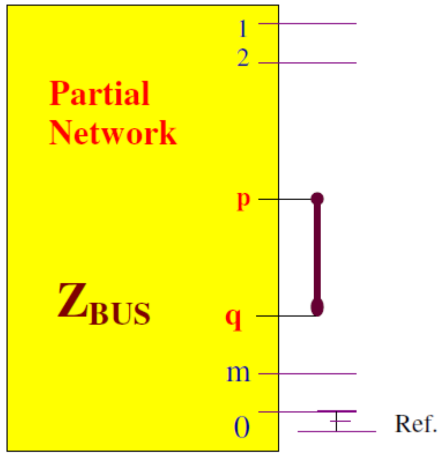


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix. If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

5.1.1. Addition of a Branch

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \tag{11}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector ypq-rs is not equal to zero and **Zij= Zji** " i,j=1,2,…m,q (12)

To find Zqi:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

Ek = Zki Ii = Zki " k = 1, 2,…i ....p,….m, q (13)

Hence, Eq = Zqi ; Ep = Zpi ………

Also, Eq=Ep -vpq ; so that Zqi = Zpi - vpq " i =1, 2,…i…....p,….m, _q (14)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \qquad (15)$$

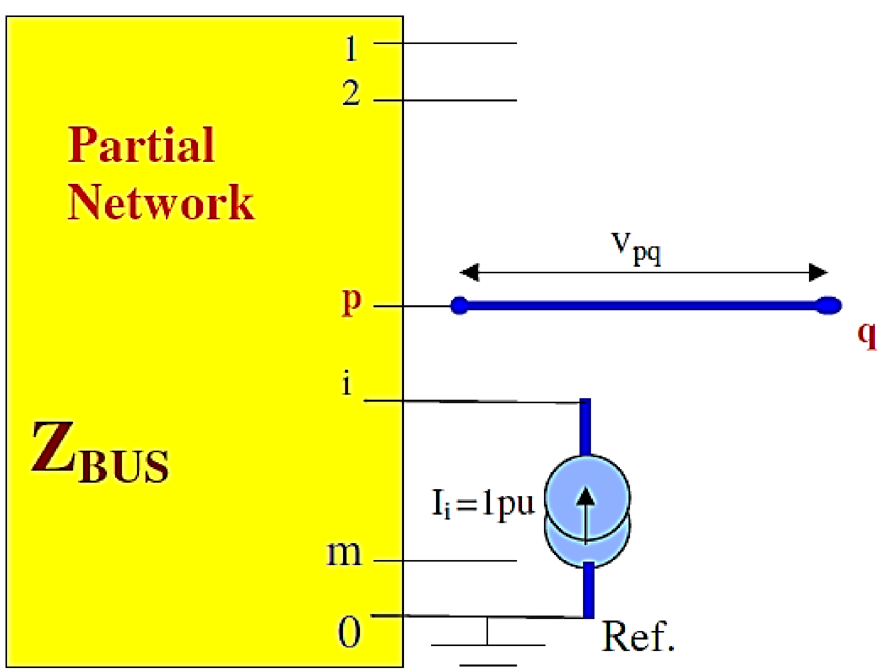


Fig.2 Calculation for Zqi

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq}v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\dfrac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2.....m; i \neq q \tag{18}$$

<u>To find zqq:</u>

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq}$ $\quad \forall$ k = 1, 2,…i…....p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq}v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \dfrac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{pi} = 0 \qquad i = 1,2......m : i \neq q$$

And $$Z_{pq} = 0.$$

Hence, from (18) (22) $$Z_{qi} = 0 \qquad i = 1,2......m; i \neq q$$

And $$Z_{qq} = z_{pq,pq} \qquad \backslash \quad (23)$$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \tag{24}$$

5.1.2. Addition of a Link

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \tag{25}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

$$\text{Vector } y_{pq\text{-}rs} \text{ is not equal to zero and } Z_{ij} = Z_{ji} \quad \forall\ i,j=1,2,\ldots m,l. \tag{26}$$

To find Zli:

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, el in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki}\, I_i = Z_{ki} \qquad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots.m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq} \quad \forall\ i=1,2,\ldots i\ldots.p,\ldots q,\ldots.m, \neq l$ (28)

To find Vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \tag{29}$$

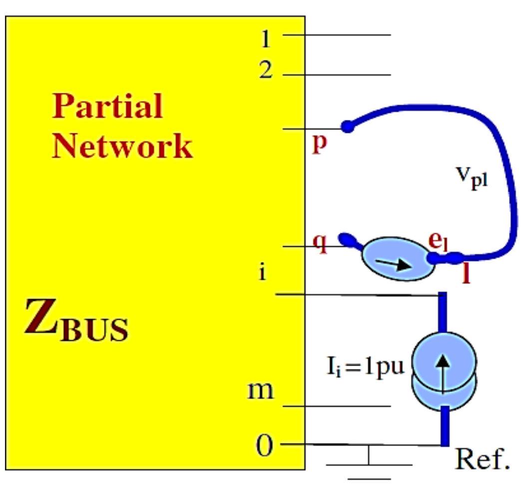


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,rs}\bar{v}_{rs} = 0 \tag{30}$$

Solving, $v_{pl} = -\dfrac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{31}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \tag{32}$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2.....m; i \neq l \tag{33}$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu. As before, we have the relations as under:

$$E_k = Z_{kl}\, I_l = Z_{kl} \qquad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots q,\ldots .m, l \tag{34}$$

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \quad (38)$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \quad (39)$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of *pq rs y* , are zero. Further, if *p* is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$\begin{aligned} Z_{ll} &= Z_{pl} - Z_{ql} - z_{pq,pq} \\ &= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq,pq} \end{aligned} \quad (40)$$

Modification of Zbus for Network Changes

An element which is not coupled to any other element can be removed easily. The Zbus is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The Zbus is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Zbus is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

5.1.3. Examples on ZBUS building

Example 1: For the positive sequence network data shown in table below, obtain ZBUS by building procedure.

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

Solution:

The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

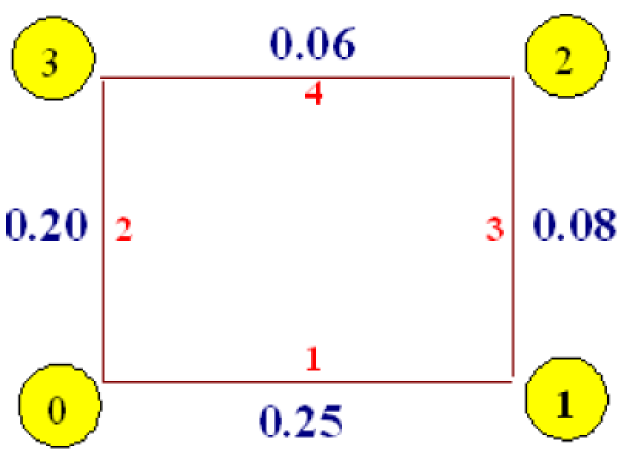
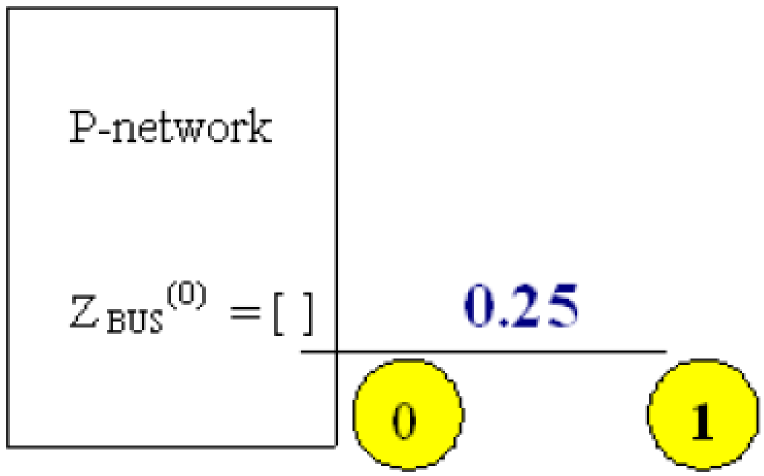


Fig. E1: Example System

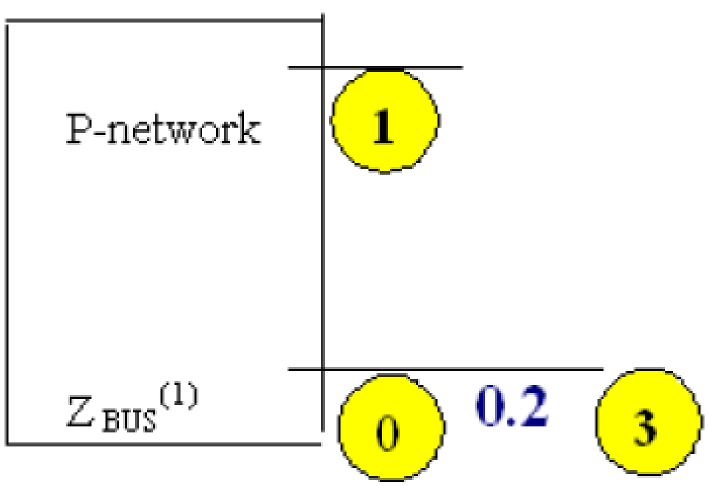
Consider building ZBUS as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



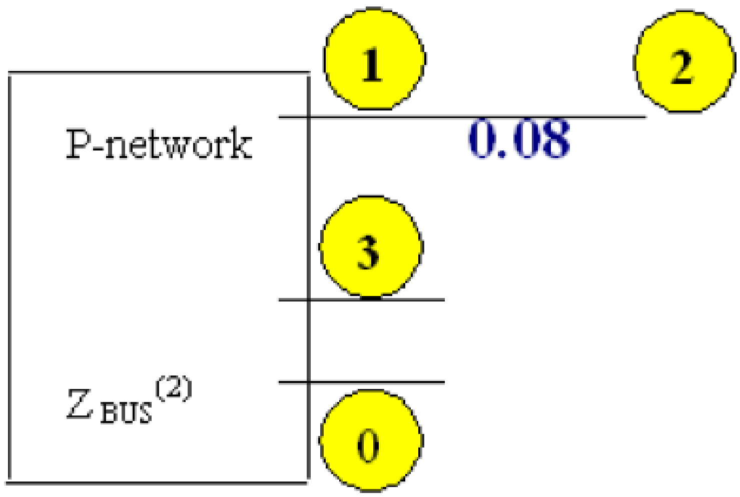
$$Z_{BUS}^{(1)} = \begin{array}{cc} & 1 \\ 1 & \boxed{0.25} \end{array}$$

Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



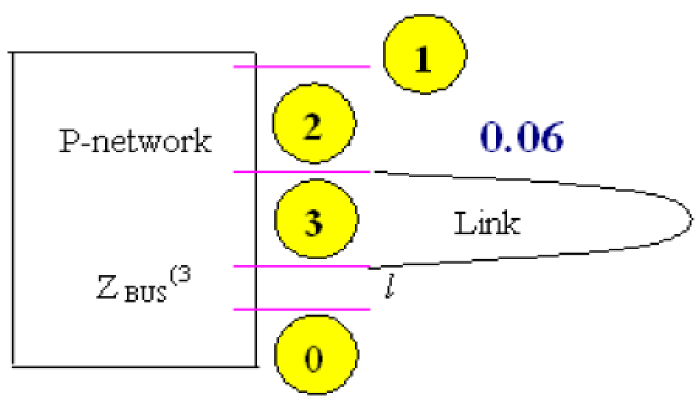
$$Z_{BUS}^{(2)} = \begin{array}{c|cc|} & 1 & 3 \\ \hline 1 & 0.25 & 0 \\ 3 & 0 & 0.2 \\ \hline \end{array}$$

Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node- 1 (p=1). (Case-b), as shown in the partial network;



$$Z_{BUS}^{(3)} = \begin{array}{c|ccc} & 1 & 3 & 2 \\ \hline 1 & 0.25 & 0 & 0.25 \\ 3 & 0 & 0.2 & 0 \\ 2 & 0.25 & 0 & 0.33 \end{array}$$

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;

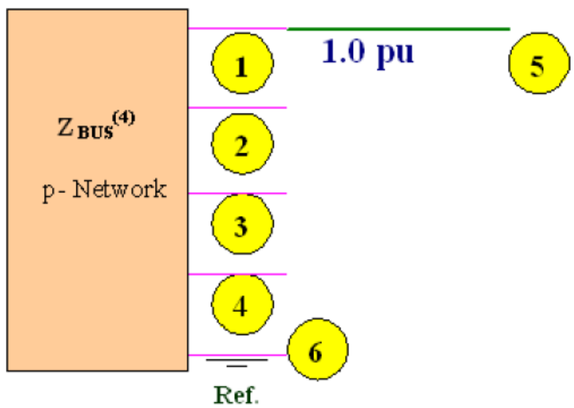


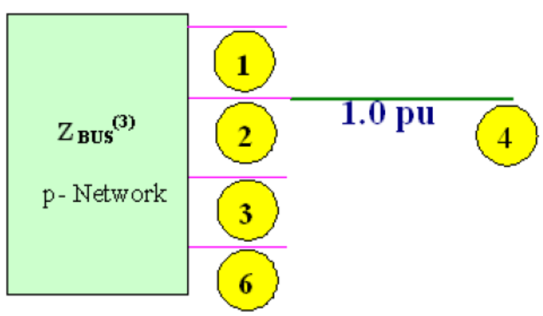
$$Z_{BUS}^{(4)} = \begin{array}{c|cccc} & 1 & 3 & 2 & l \\ \hline 1 & 0.25 & 0 & 0.25 & 0.25 \\ 3 & 0 & 0.2 & 0 & -0.2 \\ 2 & 0.25 & 0 & 0.33 & 0.33 \\ l & 0.25 & -0.2 & 0.33 & 0.59 \end{array}$$

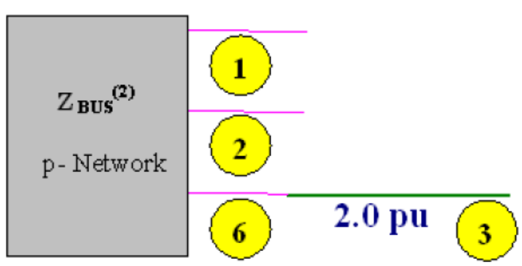
The fictitious node l is eliminated further to arrive at the final impedance matrix as under:

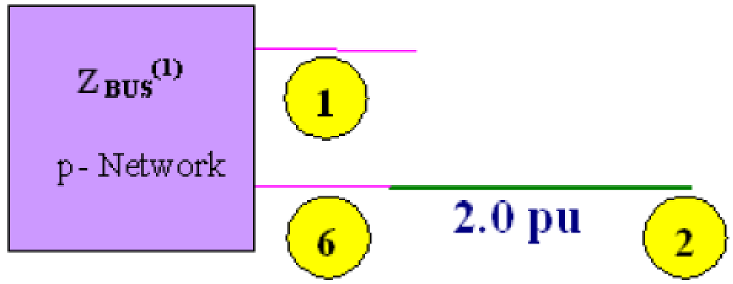
$$Z_{BUS}^{(final)} = \begin{array}{c|ccc} & 1 & 3 & 2 \\ \hline 1 & 0.1441 & 0.0847 & 0.1100 \\ 3 & 0.0847 & 0.1322 & 0.1120 \\ 2 & 0.1100 & 0.1120 & 0.1454 \end{array}$$

$$Z_{BUS} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 0 & 2 & 0 & 0 \\ 4 & 0 & 2 & 0 & 3 & 0 \\ 5 & 2 & 0 & 0 & 0 & 3 \end{array}$$

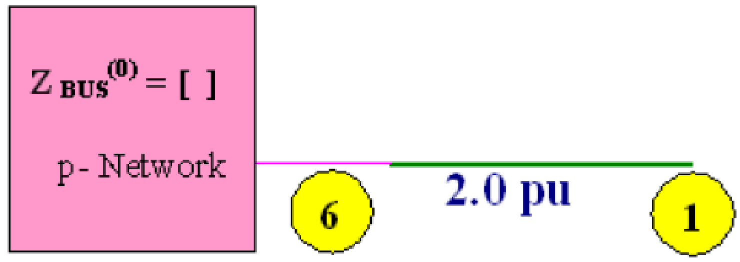




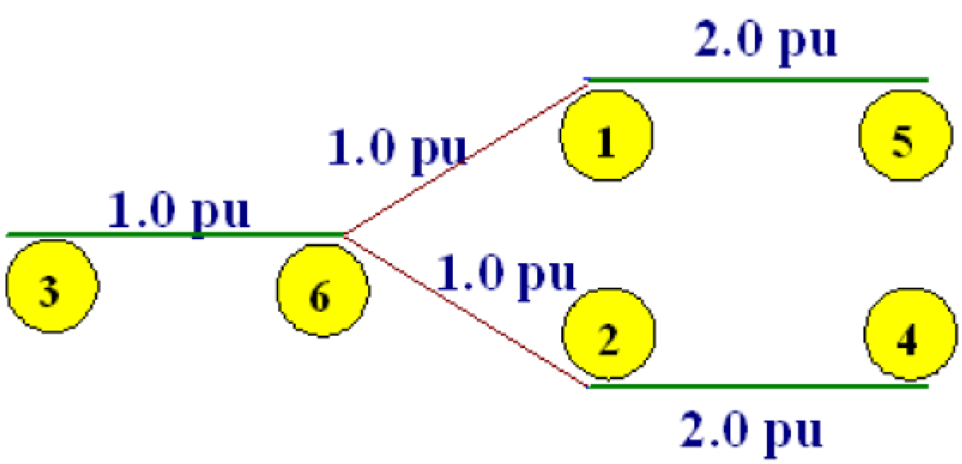




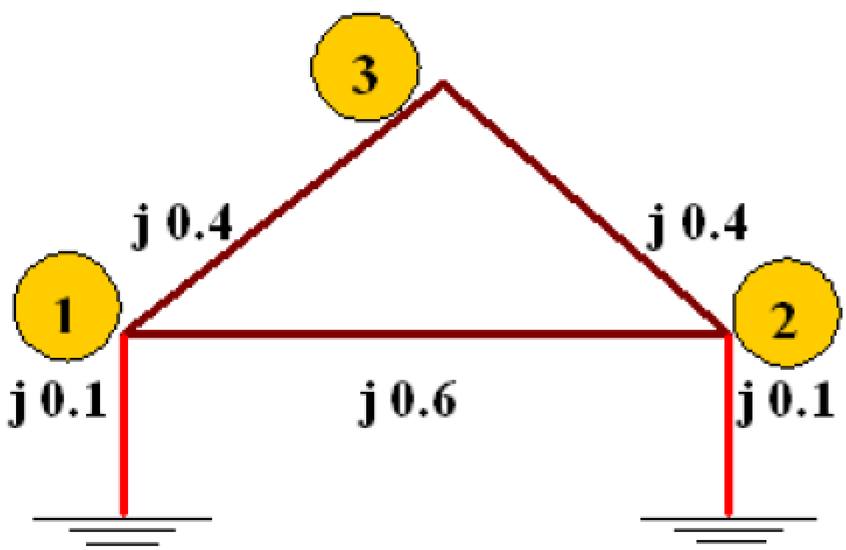
$Z_{BUS}^{(1)}$
p- Network
1
6
2.0 pu
2



$Z_{BUS}^{(0)}$ = [ ]
p- Network
6
2.0 pu
1



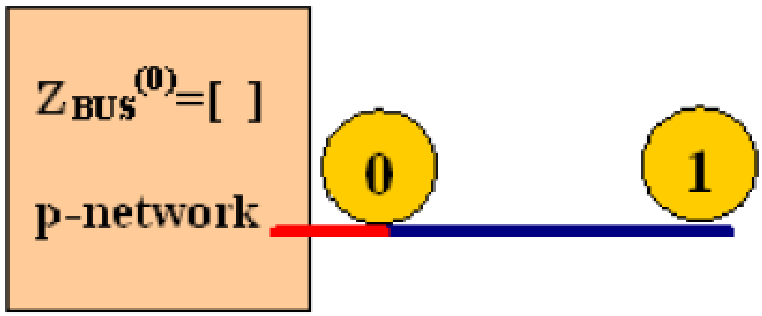
2.0 pu
1.0 pu
1
5
1.0 pu
3
6
1.0 pu
2
4
2.0 pu



3
j 0.4
j 0.4
1
2
j 0.1
j 0.6
j 0.1

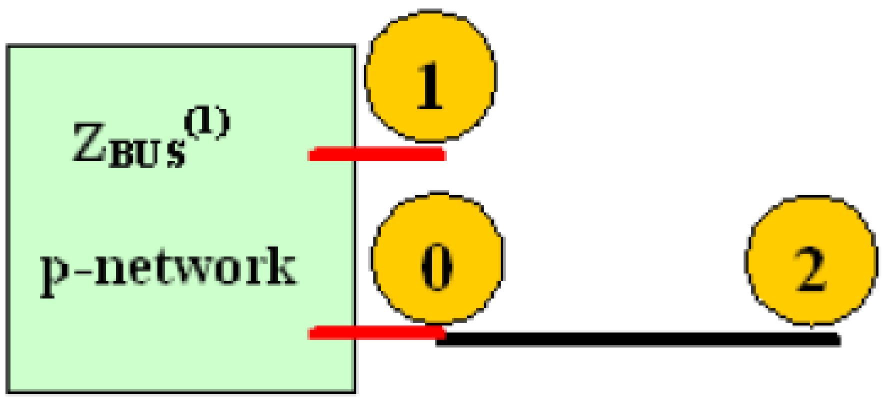
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



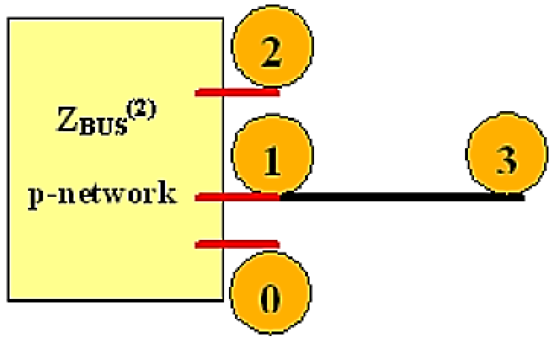
$$Z_{bus}^{(1)} = \begin{array}{cc} & 1 \\ 1 & \begin{bmatrix} j0.1 \end{bmatrix} \end{array}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



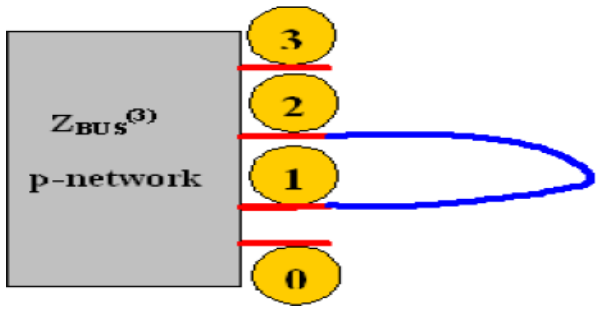
$$Z_{Bus} = \begin{array}{ccc} & 1 & 2 \\ \begin{array}{c} 1 \\ 2 \end{array} & \multicolumn{2}{c}{\begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix}} \end{array}$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$
\begin{array}{c|c}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
Z_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix}
\end{array}
$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



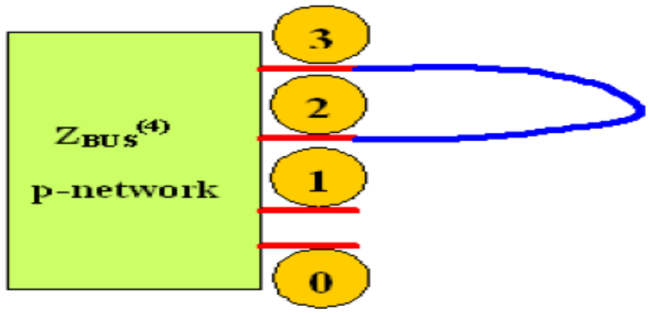
$$
\begin{array}{cc}
 & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\
Z_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & \begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix}
\end{array}
$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} / Y_{nn}} \qquad \forall\ i,j = 1,2,3.$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
Z_{bus} = & \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix}
\end{array}
$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$
$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$
$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$
$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node l is eliminated as shown in the previous step:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can be also be obtained by the rule of inspection to arrive at the same answer.

Power System Stability

5.2. Numerical Solution of Swing Equation by Point by Point method

There are several sophisticated methods for solving the swing equation. The step-by-step or point-by-point method is conventional, approximate but well tried and proven method. This method determines the changes in the rotor angular position during a short interval of time.

Consider the swing equation:

$$M\frac{d^2\delta}{dt^2} = P_S - P_{max}\sin\delta = P_A$$

The solution δ(t) is obtained at discrete intervals of time with interval spread of Δt uniform throughout.

Accelerating power, PA and change in speed, which are continuous function of time and are described as below,

1. The accelerating power PA computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered, as illustrated in Fig.1.

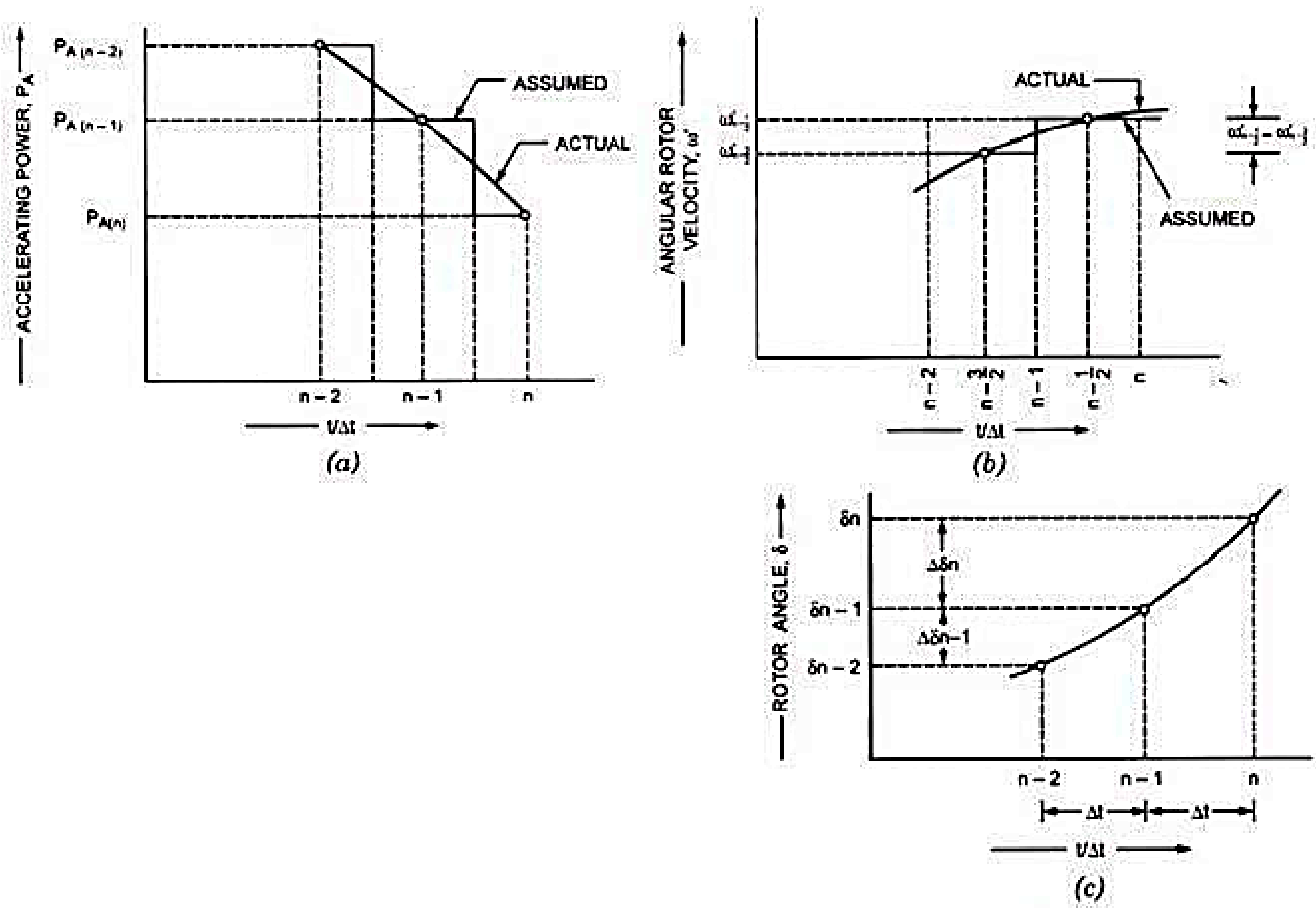


Fig. 1

2. The angular rotor velocity ω', i.e., dδ/dt (over and above synchronous velocity ω0) is assumed to remain constant throughout any interval at the value computed for the middle of the interval, as illustrated in Fig.1.

In Fig.1 the numbering on t/Δt axis pertains to the end of intervals.

The equation for accelerating power at the end of the (n – 1)th interval or for nth interval can be written as

$$P_{A(n-1)} = P_S - P_{max} \sin \delta_{n-1}$$

where δ_{n-1} has been earlier calculated.

The change in velocity caused due to $P_{A(n-1)}$ assumed to remain constant over Δt from (n – 3/2) to (n – 1/2),

$$\Delta\omega'_{n-1/2} = \omega'_{n-1/2} - \omega'_{n-3/2}$$

$$= \frac{\Delta t}{M} P_{A(n-1)}$$

The change in rotor angle δ during (n – 1)th interval,

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t \omega'_{n-\frac{3}{2}}$$

and during the nth interval, $\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t \omega'_{n-\frac{1}{2}}$

Subtracting Eq. (7.55) from Eq. (7.56) we have

$$\Delta\delta_n - \Delta\delta_{n-1} = \Delta t\left(\omega'_{n-\frac{1}{2}} - \omega'_{n-\frac{3}{2}}\right) = \frac{(\Delta t)^2}{M} P_{A(n-1)} \qquad \because \text{ from Eq. ( )}$$

$$\omega'_{n-\frac{1}{2}} - \omega'_{n-\frac{3}{2}} = \frac{(\Delta t)}{M} P_{A(n-1)}$$

$$\text{or } \Delta\delta_n = \Delta\delta_{n-1} + \frac{P_{A(n-1)}}{M}(\Delta t)^2$$

$$\therefore \delta_n = \delta_{n-1} + \Delta\delta_n$$

The above process of computation is repeated to obtain $P_{A(n)}$, Δδn + 1 and δn + 1. The time solution in discrete form is thus carried out over the desired length of time, usually 0.05 second. Actual swing curve can be plotted by drawing a smooth curve through discrete values, as shown in Fig.1.

The accuracy of the solution depends upon the time duration of the intervals. As the time interval is reduced the computed swing curve approaches the true. Usually Δt = 0.05 second provides good accuracy in results.

The occurrence or removal of a fault or initiation of any switching action causes a discontinuity in accelerating power.

There are three possibilities of occurrence of discontinuity:

(i) The discontinuity occurs at the beginning of the nth interval,

(ii) The discontinuity occurs at the middle of an interval.

(iii) The discontinuity occurs at some time other than the beginning or the middle of an interval.

In the first case, the average of the values of accelerating power P_A before and after discontinuity should be used. Thus in determining the increment of angle occurring during the first interval after the occurrence of fault at t = 0, above becomes

$\Delta\delta i = P_{A\circ}+ /2M (\Delta t)2$

whereas $P_{A\circ}+$ is the accelerating power immediately after the occurrence of the fault. Since immediately before the occurrence of the fault the system is in steady state, $P_{A\circ} - = 0$ and δ_0 is of known value.

If the fault is cleared at the beginning of the nth interval, in calculation for this interval the value of

$P_{A(n-1)}$ should be taken as

$$\frac{1}{2}\left[P_{A(n-1)^-} + P_{A(n-1)^+}\right]$$

where $P_{A\,(n-1)}$- is the accelerating power immediately before clearing and $P_{A\,(n-1)}$+ is that immediately after clearing the fault.

In second case, i.e., when the discontinuity occurs at the middle of an interval, no special procedure is required. The increment of the angle during such an interval is computed, as usual, from the value of P_A at the beginning of the interval, i.e.,

P_A = Ps – output during the fault

Where-as at the beginning of the interval following clearing of the fault, P_A is given as

P_A = Ps – output after clearance of fault.

To compute accelerating power P_A in the third case, a weighted average value of P_A before and after the discontinuity may be used. It is found in practice that such a precise computation of accelerating power P_A is not required as the time interval used in computation is so short that it is sufficiently accurate to consider the discontinuity to occur at the beginning or at the middle of an interval and accelerating power P_A is computed as outlined above in the first two cases.

5.3. Range-Kutta Method

In Range - Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation, to replace a truncated Taylor's series expansion. The formulae for the Runge - Kutta fourth order approximation, for solution of two simultaneous differential equations are given below;

$$\text{Given } \frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x(x_0, y_0, t_0)\, h$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

$$l_1 = f_y(x_0, y_0, t_0)\, h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max} \sin\delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and a step size of Δt the formulae are as follows

$$k_1 = \omega_0 \,\Delta t$$

$$l_1 = \left[\frac{P_m - P_{max} \sin\delta_0}{M}\right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\omega_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_2}{2}\right)}{M}\right] \Delta t$$

$$k_4 = (\omega_0 + l_3)\,\Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + k_3\right)}{M}\right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6}\,[l_1 + 2l_2 + 2l_3 + l_4]$$

Example

Obtain the swing curve for previous example using Runge - Kutta method.

Solution:

$$\delta_0 = 27.82^0 = 0.485 \text{ rad.}$$

$$\omega_0 = 0.0 \text{ rad / sec. (at } t = 0^-)$$

Choosing a step size of 0.05 s, the coefficient k_1, k_2, k_3, k_4 and l_1, l_2, l_3, and l_4 are calculated for each time step. The values of δ and ω are then updated. Table a gives the coefficient for different time steps. Table b gives the starting values δ_0, ω_0 for a time step and the updated values δ_1, ω_1 obtained by Runge - Kutta method. The updated values are used as initial values for the next time step and process continued. Calculations are illustrated for the time step t = 0.2 s.

$$\delta_0 = 0.756$$

$$M = 0.0331\ s^2 / \text{rad}$$

$$\omega_0 = 2.067$$

$$P_m = 0.8$$

$$P_{max} = 1.333 \text{ (after fault is cleared)}$$

$$k_1 = 2.067 \times 0.05 = 0.103$$

$$l_1 = \left[\frac{0.8 - 1.333 \sin(0.756)}{0.0331}\right] \times 0.05 = -0.173$$

$$k_2 = \left[2.067 - \frac{0.173}{2}\right] 0.05 = 0.099$$

$$l_2 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \frac{0.103}{2}\right)}{0.0331}\right] \times 0.05 = -0.\ 246$$

$$k_3 = \left[2.067 - \frac{0.246}{2}\right] 0.05 = 0.097$$

$$l_3 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \frac{0.099}{2}\right)}{0.0331}\right] \times 0.05 = -0.\ 244$$

$$k_4 = (2.067 - 0.244)\ 0.05 = 0.091$$

$$l_4 = \left[\frac{0.8 - 1.333 \sin(0.756 + 0.097)}{0.0331}\right] \times 0.05 = -0.\ 308$$

$$\delta_1 = 0.756 + \frac{1}{6}\ [0.103 + 2 \times 0.099 + 2 \times 0.097 + 0.091] = 0.854$$

$$\omega_1 = 2.067 + \frac{1}{6}\ [-0.173 + 2\times -0.246 + 2\times -0.244 - 0.308] = 1.823$$

Now $\delta = 0.854$ and $\omega = 1.823$ are used as initial values for the next time step. The computations have been rounded off to three digits. Greater accuracy is obtained by reducing the step size.

Table a : Coefficients in Runge - Kutta method

T	k_1	l_1	k_2	l_2	k_3	l_3	K_4	l_4
0.0	0.0	0.764	0.019	0.764	0.019	0.757	0.038	0.749
0.05	0.031	0.749	0.056	0.736	0.056	0.736	0.075	0.703
0.10	0.075	0.704	0.092	0.674	0.091	0.667	0.108	0.632
0.15	0.108	– 0.010	0.108	– 0.094	0.106	– 0.095	0.103	– 0.173
0.20	0.103	– 0.173	0.099	– 0.246	0.097	– 0.244	0.091	– 0.308
8.25	0.091	– 0.309	0.083	– 0.368	0.082	– 0.363	0.073	– 0.413
0.30	0.073	– 0.413	0.063	– 0.455	0.061	– 0.450	0.050	– 0.480
0.35	0.050	– 0.483	0.038	– 0.510	0.037	– 0.504	0.025	– 0.523
0.40	0.025	– 0.523	0.012	– 0.536	0.011	– 0.529	– 0.001	– 0.534
0.45	– 0. 001	– 0.534	– 0.015	– 0.533	– 0.015	– 0.526	– 0.027	– 0.519
0.50	– 0.028	– 0.519	– 0.040	– 0.504	– 0.040	– 0.498	– 0.053	– 0.476

Table b: δ, ω computations by Runge - Kutta method

t (sec)	P_{max} (pu)	δ_0 (rad)	ω_0 rad/sec	δ_1 rad	ω_1 rad/sec	δ_1 deg
0^-	1.714	0.485	0.0			
0^+	0.630	0.485	0.0	0.504	0.759	28.87
0.05	0.630	0.504	0.756	0.559	1.492	32.03
0.10	0.630	0.559	1.492	0.650	2.161	37.24
0.15	1.333	0.650	2.161	0.756	2.067	43.32
0.20	1.333	0.756	2.067	0.854	1.823	48.93

0.25	1.333	0.854	1.823	0.936	1.459	53.63
0.30	1.333	0.936	1.459	0.998	1.008	57.18
0.35	1.333	0.998	1.008	1.035	0.502	59.30
0.40	1.333	1.035	0.502	1.046	– 0.029	59.93
0.45	1.333	1.046	– 0.029	1.031	– 0.557	59.07
0.50	1.333	1.031	– 0.557	0.990	– 1.057	56.72

Note: δ_0, ω_0 indicate values at beginning of interval and δ_1, ω_1 at end of interval. The fault is cleared at 0.125 seconds. $\therefore$ $P_{max} = 0.63$ at $t = 0.1$ sec and $P_{max} = 1.333$ at $t = 0.15$ sec, since fault is already cleared at that time. The swing curves obtained from modified Euler's method and Runge - Kutta method are shown in Fig. It can be seen that the two methods yield very close results.

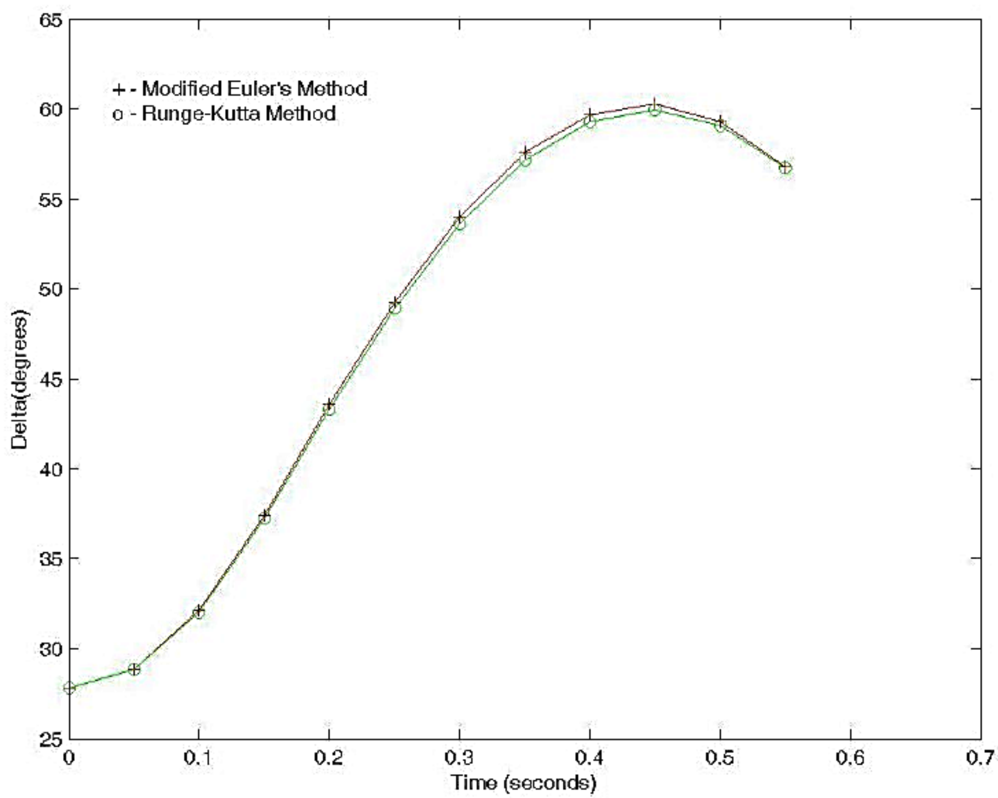


Fig: : Swing curves with Modified Euler' and Runge-Kutta methods